

# What Chances Could Not Be

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### ABSTRACT

The chance of a physical event is the objective, single-case probability that it will occur. In probabilistic physical theories like quantum mechanics, the chances of physical events play the formal role that the values of physical quantities play in classical (deterministic) physics, and there is a temptation to regard them on the model of the latter as describing intrinsic properties of the systems to which they are assigned.

I argue that this understanding of chances in quantum mechanics, despite being a part of the orthodox interpretation of the theory and the most prevalent view in the physical community, is incompatible with a very wide range of metaphysical views about the nature of chance. The options that remain are unlikely to be attractive to scientists and scientifically minded philosophers.

**1** *Introduction*

**2** *Underminability*

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### **1 Introduction**

Philosophers of science do not like to think of themselves as metaphysicians. Their work might have something to offer the metaphysicians but proceeds without their input, and is so much the better for doing so. When a philosopher of science asks what the laws of nature are, for example, he is asking a question that can be answered by presenting the equations of the best physical theories, one that does not presuppose a particular answer to the *deep* debate which divides his more speculative brethren. Or so the story goes. In at least one very important case, I will argue, this assumption of autonomy from metaphysics is mistaken. The interpretation of physical probability in quantum mechanics *cannot*

proceed without choosing sides (and, on what passes for the standard view, not choosing very well) in the metaphysical debate about the nature of chance.

The chance of a given physical event, for instance the chance that an electron in state  $S_i$  will be deflected up when passed through a non-uniform magnetic field with gradient along the z-axis, is the objective single-case probability that the event will occur. In quantum mechanics the chances play roughly the formal role that the values of basic quantities play in classical physics; they are represented by real numbers assigned to localized systems and evolve over time. Hence it is natural to construe them on the model of the values of the basic physical quantities as intrinsic properties pertaining to a system at a time.<sup>1</sup> Certain accounts of the nature of objective chance, however, are incompatible with the construal. In particular, any view of chance which allows for underminable statements about chance, i.e. statements about the chances that obtain at some time  $t$  which are incompatible with certain post- $t$  histories. I will remain neutral on the question of whether this is bad news for the notion that the chances are intrinsic or bad news for the view that the chances supervene on (non-chance) history, but by arguing against the combination of the two views, and by showing how many views of chance are infected by underminability, I want to emphasize that one *cannot*—and ought not try to—remain innocent of metaphysics while thinking about the interpretation of chance in physics.

## 2 Underminability

I will follow David Lewis in using the term ‘theory of chance of  $w$ ’, written  $T_w$ , to refer to the set of conditionals which give the chances of future events at any time on the basis of preceding history, reserving the term ‘view of chance’ for the philosophical analysis which gives the theory of chance for any particular world. I will assume a world to have the structure of a manifold of events, and the complete description of a world to be given by an assignment of values of basic quantities to all points. By ‘pre- $t$  history’, I mean an assignment of basic quantities to points up until and including a time  $t$ , and by ‘post- $t$  history’ an assignment of basic quantities to points after  $t$ . It is assumed that the chances are themselves not among the basic quantities, so when I talk about pre- and post- $t$  histories, I mean these to be characterized without reference to the chances. Pre- $t$  history

<sup>1</sup> The view is succinctly expressed by Teller: ‘the closest quantum theoretical analogue [to the classical values of physical quantities] are the theory’s probabilities, or, more generally, its expectation values’ (Teller [1995], p. 153).

does not include, for example, the fact that the chance of a post- $t$  coin toss coming out heads is  $1/2$ .<sup>2</sup>

A statement  $S$  about the  $t$ -chances is *underminable* just in case there are possible post- $t$  histories with which it is incompatible. In any such case, the pre- $t$  distribution of chances *contains information about post- $t$  history*. To see how this works, consider a particularly simple view of chance, the so-called actual frequentist view according to which the probability of an event  $A$  in a reference class  $B$  is the relative frequency of occurrences of  $A$  within  $B$ .<sup>3,4</sup> So, for example, the chance that an electron is deflected up when passed through a non-uniform field with a gradient along the  $z$ -axis, if we let  $N^n(A)$  = the number of electrons in state  $S_i$  actually passed through such a field and  $N^n(A.B)$  = the number of such electrons which are actually deflected upwards, is  $N^n(A.B)/N^n(A)$ . On such a view,

- (i) worlds with different histories will sometimes have different theories of chance. This follows from the fact that frequencies vary from world to world, and
- (ii) the theory of chance of a world  $w$  will not supervene on any *finite initial segment* of  $w$ 's history. This follows from the absence of any guarantee that short-range frequencies will reflect long-range frequencies.<sup>5</sup>

These two things together entail that for *any* world  $w$  and *any* time  $t$ , there will be post- $t$  histories for  $w$  compatible with its pre- $t$  history but incompatible with the  $t$ -chances, i.e. worlds with histories which are the same as  $w$ 's up until  $t$  and which differ thereafter only in the outcomes of chancy events. Since chances reflect long-range frequencies of chancy events, and since differences in post- $t$  frequencies of the right sort make for differences in  $t$ -chances, among these there are worlds which differ from  $w$  with respect to the  $t$ -chances. These are worlds in which the post- $t$  frequencies depart significantly enough and in the right way from those

<sup>2</sup> This is, at this point, just a point of terminology, and not meant to beg the question against the view that chance is a basic physical quantity in its own right, which I shall take up later.

<sup>3</sup> Venn held a very strict form of actual frequentism. Reichenbach also said some things suggestive of actual frequentism, though he ended up defending something more like hypothetical frequentism. Cramer, Hempel, and Putnam all at one time pursued actual frequentist accounts.

<sup>4</sup> If the history of the world is finite and if systems get only 'just so small' (i.e. are of finite size), then all reference classes are finite. If we allow point-sized systems, however, there might be infinite sequences, and in such cases the chances are identified with limit of the relative frequency—if such there be—of  $A$ 's among a sequence of  $B$ 's.

<sup>5</sup> Actually, (i) follows from (ii), since if the world does not supervene on any finite initial segment of the history of the world, it cannot be a constant function, and if it is not a constant function, then (i) is true.

of  $w$ .<sup>6</sup> So on the actual frequentist account, for any world  $w$  and any time  $t$ , it is possible relative to pre- $t$  history that  $w$  will follow a post- $t$  course which is incompatible with the  $t$ -chances at  $w$  being what they in fact are, which is to say that the chance distribution at any time is underminable.

### 3 The prevalence of underminability

The same thing will hold for any view of chance according to which the statement that the  $t$ -chance of some event is such-and-so is incompatible with even a single post- $t$  distribution of events, and this in turn is true of any view according to which the chance distribution at a world supervenes on its total history but not any finite initial segment of that history. Suppose this is not so, i.e. suppose that according to some such view, the chances *cannot* be undermined. This means that the  $t$ -chances at any world  $w$  are compatible with all post- $t$  histories for  $w$ , and this in turn means that for any  $t$ , the theory of chance is the same at all worlds which share pre- $t$  history, no matter what happens after  $t$ . But *this* just means, contrary to our hypothesis, that the theory of chance supervenes on the segment of  $w$ 's history which precedes  $t$ . So, on any view according to which  $T_w$  supervenes on total but not any initial segment of history, chances will be underminable.

The manner in which the chances are undermined will vary from view to view, but *on any view according to which a statement about the  $t$ -chances places some constraint—no matter how weak—on post- $t$  history, the statement will be undermined by a history which violates the constraint*. Actual frequentist views of chance fall most obviously into this class, and the way in which the  $t$ -chances are undermined on such a view is particularly easy to see. Other cases, for instance those in which the chances are identified with counterfactual relative frequencies of one or another sort, will suffer the same fate so long as the truth values of the counterfactuals describing those relative frequencies *themselves* supervene on  $w$ 's history. No matter how complicated the connection between the actual world and the relevant counterfactuals, insofar as the chances are determined by the counterfactual frequencies and insofar as the counterfactual frequencies are determined by actual history (and not just the pre- $t$  segment of it), there must be ways for post- $t$  history to go which are incompatible with the truth of the relevant counterfactuals, and these are the post- $t$  futures which

<sup>6</sup> Since worlds which share  $w$ 's past and present but follow an undermining future with respect to the  $t$ -chances at  $w$  differ from  $w$  only in the outcomes of chancy events, their  $t$ -chance at  $w$  is non-zero. So, not only are undermining futures metaphysically possible relative to pre- $t$  history,  $T_w$  itself assigns them some positive chance of occurrence. It is *this* fact which made undermining futures so problematic for the formulation of David Lewis's Principal Principle in Lewis [1986].

undermine the t-chances. One way in which the counterfactuals may turn out to supervene on total history is by being determined by the laws of the world at which they are assessed. To disallow undermining altogether, an account of chance must completely sever the link with actual history; worlds with identical histories must sometimes differ with respect to the chances.

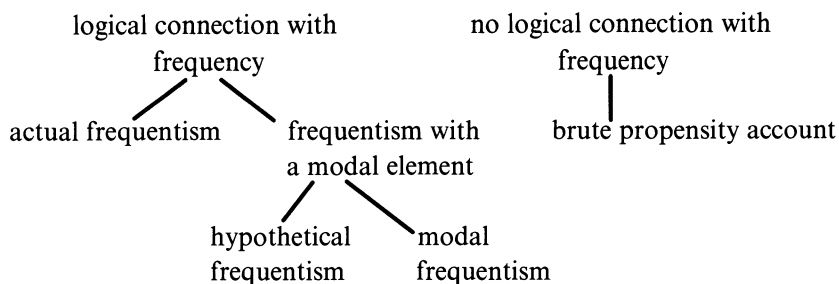
#### 4 Views of chance

The most natural way of classifying accounts of chance, or as it is often called in this context ‘objective probability’, is according to whether they recognize an analytic connection between chance and frequency.<sup>7</sup> On the brute propensity account (whose most influential proponent is Giere) probability is an objective quality distinct from actual frequency; there is no necessary connection whatsoever between the two. Among those accounts which do recognize an analytic connection between chance and frequency, there are those which identify chance with frequencies in an actual reference class, and those which identify chances with frequencies in one or another counterfactual reference class. Of the latter, the hypothetical frequentism of Reichenbach, which identifies chances with frequencies in an infinite run from which the actual is considered a random sample (and with which the actual is identical if it is itself infinite), is closest in spirit to actual frequentism, and the modal frequency view of van Fraassen, which identifies chances with frequencies in a family of models of ideal experiments, is the most recent and sophisticated view sprung from frequentist soil. Here are the definitions:

- (i) Brute propensity view: probability is an objective quality distinct from actual frequency. Even in an infinite sequence of experiments under correct fixed conditions, the relative frequency and the probability need not coincide.
- (ii) Actual frequentism: the probability of an event A in a reference class B is the relative frequency of occurrences of A within B (or, if there are infinitely many occurrences of both, the limit of the relative frequency of A among a sequence of B’s).
- (iii) Hypothetical frequentism: the probability of an event A in a reference class B is what the limit of the relative frequency of A’s among the B’s would be hypothetically if the actual (finite) sequence were extended to an infinite sequence.
- (iv) Modal frequentism: the probability of an event A equals the relative frequency with which it would occur, were a suitably designed experiment performed often enough under suitable conditions.

<sup>7</sup> For a recent survey of views of chance see Howson [1995], also van Fraassen [1979].

The views are related as follows:



The problem for brute propensity theorists is to wring from their account the conclusion that beliefs about probabilities are a rational guide to expectations about relative frequencies, a problem that seems to some rather like trying to squeeze water from a stone. Actual frequentist accounts also have problems, some of them of a general philosophical character, some of them stemming from the fact that proportions of finite reference classes do not in general have the mathematical properties of probability measures. Hypothetical and modal frequentist views solve some of these, but at the price of a heavy dose of metaphysics, and they acquire problems of their own into the bargain. None of these is our concern. What we want to know is according to which of these views the chances are underminable. We want to ask of each: ‘For any  $t$ , could the world go in such a way after  $t$  as to undermine the  $t$ -chances?’<sup>8</sup>

In the case of the brute propensity accounts, the answer is clearly ‘no’, and in the case of the actual frequentist accounts, we saw that the answer is just as obviously ‘yes’. With respect to the modal accounts, the answer depends on how the truth values of the relevant counterfactuals are determined. If—as most of the accounts on the books have it—counterfactuals supervene on actual history without supervening on any initial segment of it, the chances will turn out to be underminable. The upshot is that underminability might at first appear to infect just that class of views which tie the chances very closely to frequencies in *actual* reference classes, but is in fact much more prevalent; only one of the views above, the brute propensity view, is completely immune.

## 5 The chances are not intrinsic if they are underminable

I said that the underminability of a statement about the  $t$ -chances shows

<sup>8</sup> One might wonder where a David Lewis-style view, according to which chances are determined by probabilistic laws, and laws (probabilistic and otherwise) supervene on history, fits in. It is a version of hypothetical frequentism; the truth values of counterfactuals describing relative frequencies in infinite runs are determined by the laws and reflect the probabilities as the hypothetical frequentist would have it.

that it *contains information about the post- $t$  history of  $w$* ; what I meant was that it contains information which transcends what is derivable from pre- $t$  history together with  $w$ 's laws. Since there are no metaphysically necessary connections between the intrinsic properties of  $w$  at  $t$  and its intrinsic properties at a later time  $t'$ , the existence of underminable chance statements can only mean that the  $t$ -chances are not in general an intrinsic property of  $w$  at  $t$ .<sup>9</sup> We can turn this into a proper argument by noting that if  $P$  describes the intrinsic property of any individual  $a$ , then for all permutations of the intrinsic properties of individuals other than  $a$ , there is a possible world in which  $Pa$  obtains as well as one in which  $\sim Pa$  obtains. Now, what it is for objective chance to be a basic physical quantity is for its values to represent possible intrinsic properties of points. So let us assume:

- (i) that the world consists of a distribution of physical quantities over an arrangement of points, and that the objective chances obtaining at  $t$  are an intrinsic property of  $w$  at  $t$ , and
- (ii) that the chances at  $w$  supervene on the total history of  $w$ , but not any initial segment of that history. Then it follows that
- (iii) the complete statement of chances pertaining to  $w$  at  $t$  rules out certain metaphysically possible courses for the world subsequent to  $t$ , since it implies that  $w$  will not follow an undermining course.

But this shows, contrary to the assumption in (i), that the distribution of objective chances of  $w$  at  $t$  is *not* an intrinsic physical property of  $w$  at  $t$ . It should be easy to see that nothing in the argument hinges on the particular ontology of points, spatiotemporal relations, etc. It can be made, *mutatis mutandis*, taking localized material bodies and causal relations as the relevant individuals and relations, for example. So long as no metaphysically necessary connections are allowed between distinct existences and the world at  $t$  is distinct from the world at any distinct time  $t'$ , the argument will go through.

## 6 Intrinsic properties

As to the step from the claim that the distribution of chances at  $t$  is an intrinsic feature of the world at  $t$  to the claim that it is compatible with all other combinations of intrinsic properties of distinct individuals: the intrinsic features of an individual  $\hat{I}$  are usually glossed as those it possesses

<sup>9</sup> By 'basic intrinsic properties of  $w$  at  $t$ ', I just mean the physical quantities instantiated at every point on a space-like hypersurface of  $w$  corresponding to  $t$ . Since each of these points are distinct existences, the disjoint sets of them which form space-like hypersurfaces at different times are distinct existences, and so if there are no metaphysically necessary connections between distinct existences, there are no metaphysically necessary connections between the intrinsic properties of  $w$  at different times  $t$  and  $t'$ .

independently of the way the rest of the world is. This is trivially falsified for any property whatsoever if we include in the description of ‘the way the rest of the world is’ relational properties it bears to  $\hat{I}$  (consider the property ‘is such as to bear such-and-such a relation to an individual [ $\hat{I}$ ] with such and such intrinsic character’), so we should revise the gloss as follows: the intrinsic features of  $\hat{I}$  are those it possesses independently of the intrinsic features of all individuals distinct from  $\hat{I}$ . And we can make this precise by a principle of the sort I proposed:  $P$  is an intrinsic property of  $\mathbf{a}$  iff for all distinct individuals  $\mathbf{b}$ , and all intrinsic properties  $S$  of such individuals, there are possible worlds in which each of the combinations  $\mathbf{Pa} \ \& \ \mathbf{Sb}$ ,  $\mathbf{Pa} \ \& \ \sim \mathbf{Sb}$ ,  $\sim \mathbf{Pa} \ \& \ \mathbf{Sb}$ , and  $\sim \mathbf{Pa} \ \& \ \sim \mathbf{Sb}$  obtain.<sup>10</sup>

The principle relates the claims that  $I = \{a, b, c, \dots\}$  is a set of distinct individuals,  $J = \{R, S, \dots\}$  a set of physical quantities the values of which correspond to intrinsic properties of such individuals, and  $W = \{w_1, w_2, w_3, \dots\}$  the set of metaphysically possible worlds. We can use it—as David Lewis does in *On The Plurality of Worlds*—to delimit the set of possible worlds on the basis of a choice of individuals and their intrinsic properties. As such it is an expression of the Humean ban on necessary connections between distinct existences; it says that there is a metaphysically possible world for each mathematically possible arrangement of individuals (specification of the external relations they bear to one another) and distribution of physical quantities over them.

We can also use it, if we have some independent grip on the constitution of the set of possible worlds, to test the claim that some property is intrinsic to a given individual. Here is a simple example: we can ask whether being Princess of Wales is an intrinsic property of Lady Di. Lady Di and England are distinct existences, so there are worlds in which Lady Di exists and England is intrinsically any which way you please. In particular, there are worlds in which England has no royalty. But there are no such worlds in which Di is still Princess of Wales, so being Princess of Wales is not an intrinsic property of Di. Or we might wonder whether the property of being a widow is intrinsic to Jocasta. We notice that there are worlds in which both Jocasta and Laius exist, and indeed are married to one another, but in which Laius escapes his grisly fate at the crossroads and outlives Jocasta by a decade. There are, however, no such worlds in which Jocasta is a widow, so Jocasta’s widowhood is not intrinsic to her. The idea is that there are no metaphysically necessary connections between the intrinsic properties of distinct individuals, so one way to show that a particular property is not intrinsic to a given individual is to show that it is not compatible with any old assignment of intrinsic properties to distinct individuals.

<sup>10</sup> I can remain neutral on the question of whether this is a substantive metaphysical truth or simply an analysis of the notion of an individual.



So, on certain views of chance, the distribution of chances of future events assigned to a world  $w$  at a time  $t$  are not intrinsic to  $w$  at  $t$ , because the chances at any given time depend in part on  $w$ 's subsequent biography. They depend, that is, on the frequencies of similar events in a certain kind of ensemble (called a time ensemble) consisting of a single system—here,  $w$  as a whole—brought repeatedly into the same state. I could have just as easily shown that the chances assigned to any localized system  $s$  within  $w$  (say, the chance pertaining to an electron, that a future interaction with a measuring device will have a particular outcome) are not intrinsic to  $s$  at  $t$ . In fact, an almost identical argument will show that the chances pertaining to  $s$  at  $t$  depend on frequencies of similar events in an ensemble of distinct systems in the same state at a single time (a so-called space ensemble). The chances pertaining to a system at a time are partly a matter of facts about the system at other times, and partly a matter of facts about other systems at the same time. On any view of chance according to which there is a metaphysically necessary connection between the chance of a given type of event and frequencies of similar events in ensembles of similar systems (both space ensembles and time ensembles), the chances—despite the fact that they are represented by numbers assigned to systems occupying localized regions in space-time—are radically unlocalized. This may not be news to anyone who has spent time thinking about chance from a metaphysical perspective, but it is *not* something which has been sufficiently appreciated in the context of discussions of chance in quantum mechanics.

## 7 Chance in quantum mechanics

The intrinsic character of any physical system is given by saying what type of system it is (listing its state-independent properties) and specifying its state. In quantum mechanics, a system's type is given by specifying the number and type of particles of which it is composed and its state is represented by a mathematical entity, a vector in a certain sort of vector space. There is a rule for calculating from its state vector, the probability that a system will behave in one or another way in a certain class of physical interactions, specifically, that it will show one or another result in a measurement of a given observable.<sup>11</sup> There is an equation which describes the evolution of an isolated system when no measurement is being carried out, and—on the orthodox or 'Copenhagen' interpretation—a rule which predicts the evolution of a system during a

<sup>11</sup> Accept uncritically for the purposes of the discussion, a division of physical interactions into measurements and non-measurements.

measurement. All of these things have names: the mathematical entity which represents the state of a system is called its *wave-function*; it describes a vector in a *Hilbert space*. The rule for calculating the expectation values for observables from the wave function is called *Born's Rule*, the equation describing evolution outside measurement contexts is *Schroedinger's equation*, and finally the rule describing state transitions during measurement is the *collapse postulate*.

Giving an interpretation of a physical theory is just what it sounds like: specifying what the theory says about the way the world is. In quantum mechanics this is a matter of understanding what it means to say that a system is in a state represented by a given vector  $\psi$ . Since we can use Born's rule to calculate probabilities for measurement results,  $\psi$  contains at the very least the information that the system will yield one or another result in a measurement of any observable with a given probability. The orthodox interpretation adds to this the assertion that the system has definite values for *all and only* observables for which it is certain to yield one or another value (i.e. those of which it is in an *eigenstate*), and the value it actually has is the value which it is certain to yield (its *eigenvalue*).<sup>12</sup> So on the orthodox interpretation, the set of truths about a system's intrinsic state at a time is exhausted by listing its state-independent properties (mass, charge, spin-type), the eigenvalues of the observables of which it is in an eigenstate, and the expectation values for all observables of which it is not in an eigenstate.

All of the preceding rumination about chance was prompted by the question of what it could possibly mean to say that the expectation values (partly) characterize the intrinsic state of a system. If what I have been saying is right, then *if* the chances are underminable, they are not intrinsic to a system at a time. In just the same way Jocasta's being a widow is not intrinsic to Jocasta because it is incompatible with the intrinsic features of other regions being any which way intrinsically (specifically, with Laius' being alive and well at the distant crossroads). The existence of underminable statements about the chances shows that they are not intrinsic to a localized system at a time since it shows that there are facts about what happens at other places and times with which they are incompatible.

## 8 What the options are

One option is to reject the notion that quantum mechanical expectation values describe intrinsic properties in their own right. We can hold instead that they merely *pick out* intrinsic properties which in fact cause the system's chance behaviour and which are in principle describable in terms that

<sup>12</sup> This is von Neumann's so-called Interpretation Rule.

make no reference to the events they are chances of. On this understanding, expectation values are avowedly instrumental descriptions of properties describable directly in microscopic terms which underlie the system's disposition to exhibit one or another outcome with certain frequencies when coupled with 'measuring apparati' of relevant sorts. The view is consistent but unattractive for the reasons that instrumentalism is unattractive in general, compounded by the fact that in this case it characterizes the content of the theory in terms that have yet to receive—despite a good deal of attention—a mathematically precise meaning, i.e. results of 'measurements' or effects on 'macroscopic' or 'classical' physical systems. Nor is it possible to offer it as a gloss of the Copenhagen interpretation, for it goes directly against the denial—which is the distinguishing characteristic of the interpretation—that any description in microscopic terms (terms that do not mention interaction with macroscopic instruments and their classical properties, or measuring devices, or anything like that) is possible. In Bohr's words:

it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms (N. Bohr, Schilpp).

Another option is to opt for a brute propensity view of chance, or to combine one or another of the modal views with an account of counterfactuals according to which the truth values of the relevant ones do not supervene on actual history. This is an uncomfortable position because the injection of metaphysics needed to sustain the latter goes against the empiricist inclinations that typically motivate the former. Not to mention the question of whether such views pass muster independently as accounts of chance and counterfactuals.

I do not see any other options, which is not to say that there are none but that more thought ought to go into understanding what they are. This much is certain; there is a conflict between a very wide class of views about chance and the standard way of thinking of chance in quantum mechanics. We need to be much more self-conscious about both in thinking about either, and philosophers of science need to engage the metaphysical debate head on.

There is one possibility which I have refrained from mentioning because it amounts to the denial that there is anything properly called chance, insofar as it is distinct from any species of subjective probability. What I have in mind is the view that chance is just epistemic probability of a particular sort: consider two systems A and B, which differ with respect to the probability pertaining to A of some future event type (say a collapse into a state  $\psi$  at  $t$ ). A either will or will not collapse into  $\psi$  at  $t$ , B either will

or will not collapse into  $\psi$  at  $t$ , and there is nothing we can now ascertain about either system (e.g. no measurement we can perform) which will decide the case for certain. The occurrence of  $e$  is, however, differently correlated with facts about  $A$  and  $B$  which we *can* determine and with respect to which they differ, e.g.  $e$ -type events occur to  $1/2$  of systems relevantly like  $A$  and only  $1/4$  of those like  $B$ . This means that we can place  $A$  and  $B$  in ensembles of systems—as alike to them as we can presently ascertain—in which the frequency of  $e$ -type events is  $1/2$  and  $1/4$ , respectively, and this is what the probabilities refer to. The difference between systems with different biographies of probabilities (so long as these assign non-maximal probabilities [i.e.  $0 < pr < 1$ ] to the same set of events), is not a difference in the properties of those systems but a difference in what we know about them.

Something about so interpreting probabilities in quantum mechanics has struck physicists as objectionably subjective. Wrongly so, in my opinion. The epistemic probability of a proposition  $p$  can be (and often is) represented as a relation between  $p$  and a given agent, but can just as well be represented as a relation between  $p$  and a set of propositions, those representing the body of information possessed by the agent in question. The  $t$ -chance of  $e$  on  $A$  can be defined as the probability of  $e$  relative to the propositions which describe  $A$ 's state at  $t$  or its pre- $t$  history, i.e. as the personal probability of an agent who has no foreknowledge but is perfectly apprised of  $A$ 's history and the laws which govern its evolution. There is nothing scientifically disreputable chances so construed, indeed it is not unlikely that many physicists have been operating with such a conception without distinguishing it carefully from the view that the  $t$ -chances pertaining to  $A$  describe objective properties of  $A$  such as mass, spin, and the like. The difference between the two views is just the difference there is in general between the view that some 'parameter'  $P$  is a redescription of those in another set  $\{P_1, \dots P_n\}$ , and the view that  $P$  is ontologically distinct from but happens to covary with  $\{P_1, \dots P_n\}$ . In either case, fixing the values of  $P_1, \dots P_n$  fixes the value of  $P$  in all *physically* possible worlds, but the value of  $P$  varies independently of those of  $P_1, \dots P_n$  in the set of *metaphysically* possible worlds only in the latter case. The value of  $P$  fails to supervene on those of  $\{P_1, \dots P_n\}$  *iff*  $P$  is ontologically distinct from  $\{P_1, \dots P_n\}$ .

Interpreted objectively, quantum probabilities describe properties ontologically independent of the system's other properties and partially characterize its intrinsic state, and it must be (metaphysically) possible for two systems—otherwise alike—to differ with respect to their chances. Interpreted epistemically, the intrinsic state of a system at  $t$  is imperfectly correlated with aspects of its post- $t$  state (the events to which it assigns

non-maximal probabilities). In this case, the probabilities reflect only uncertainty about the future of the system based on knowledge of its present state (excluding, of course, the chances), and it is impossible for two systems—otherwise intrinsically alike at  $t$ —to differ with respect to the  $t$ -chances pertaining to each. This view seems to me completely adequate to the role of probabilities in physics (though it would take a much longer paper to show this), and—of all those described—the most natural.

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