A MODEST PROPOSAL ABOUT CHANCE*

Before the year 1600 there was little uniformity in conception of natural laws. In the seventeenth century, the rise of science, Newton's mathematization of physics, and the provision of strict, deterministic laws that applied equally to the heavens and to the terrestrial realm had a profound impact on the philosophical imagination. A philosophical conception of physical law built on the example of Newtonian mechanics quickly became entrenched. Between the seventeenth and twentieth centuries, there was a great deal of philosophical interest in probabilities, but probabilities were regarded as having to do with the management of opinion, not as having a fundamental role in science.[1] Probabilities made their first appearance in an evidently ineliminable way in the laws of a fundamental theory with the advent of quantum mechanics. Quantum probabilities have come to be called "chances" in the philosophical literature, and their interpretation has been one of the central problems in philosophy of science for almost a century. Hold-outs continue to insist that there must be an underlying, probability-free replacement for quantum mechanics, and Bohmians have had some success in formulating a deterministic alternative to quantum mechanics, but contemporary physicists accept that the probabilistic character of the quantum-mechanical laws is likely to be retained in any successor theory. While physics has adjusted itself comfortably to the existence of ineliminably probabilistic laws, philosophy has not managed to arrive at a stable interpretation of chance. The difficulty is that such an interpretation must satisfy a number of constraints. These constraints appear to be partially definitive of the concept, and it proves extraordinarily difficult to meet them simultaneously.[2]

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[1] Statistical mechanics did bring probabilities into physics, but not at the fundamental level and not in a way that challenged the strict necessity of the fundamental laws.

[2] "Chance" is sometimes used to refer to any form of objective probability. I use it narrowly to refer to quantum probabilities.

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Recently, there has been a push from a number of quarters in the philosophy of physics and the special sciences to recognize a form of probability in deterministic contexts. David Albert and Barry Loewer have argued for the objective nature of statistical-mechanical probabilities.[3] Elliott Sober has argued forcefully for the irreducibility of macro-probabilities and the need to recognize such probabilities.[4] I have argued that probabilities are indispensable quite generally in the formation of expectation, confirmation, and explanation, given any constraint on knowledge of the full microstate of the universe.[5] Myrvold and Grieves, reflecting on confirmation in an Everett universe, have found a role for probabilities in confirmation that is indifferent to whether or not the setting is deterministic. There are differences in motivation and detail among these defenders of objective, nonquantum probability.[6] All of them argue, however, that we have reason independent of quantum mechanics for recognizing a form of objective probability.[7]

The probability in question takes the form of a measure $\mathrm{Pr}_G(A/B)$, where A and B are (states or properties represented by) finite volumes of phase space and Pr_G is the probability that a random pick from (systems in) B will yield a system that is in A, or on an A-bound trajectory.[8] Pr_G defines both synchronic macroscopic probabilities (for example, the probability that an emerald is green or a woman over 40 has flat feet), as well as transition probabilities (for example, the probability that a baseball hitting a window at a speed of 40 miles per hour will break it).[9] In what follows, I will show how we can use

[3] Barry Loewer, "Chance and Determinism," in Yemima Ben-Menahem and Itamar Pitowsky, eds., "The Conceptual Foundations of Statistical Physics," *Studies in the History and Philosophy of Modern Physics*, XXXII, 4 (December 2001): 609–20; and David Albert, *Time and Chance* (Cambridge: Harvard, 2000). John Pollock, *Nomic Probability and the Foundations of Induction* (New York: Oxford, 1990) is an early precursor.

[4] Elliott Sober, "Evolutionary Theory and the Reality of Macro-Probabilities," in James H. Fetzer and Ellery Eells, eds., *The Place of Probability in Science* (New York: Springer, 2010), pp. 133–61.

[5] Jenann Ismael, "Probability in Deterministic Physics," this JOURNAL, CVI, 2 (February 2009): 89–108.

[6] Or not-specifically-quantum probability. David Albert has argued that they ultimately may derive from quantum probability.

[7] To say that the probabilities are objective is to say at least that they are distinct from the subjective probabilities of the user of a theory. They are something that, to the extent that he is a good epistemic agent, *guides* his subjective probabilities.

[8] Ismael, *op. cit.*

[9] It can be derived from a distribution over initial conditions in deterministic contexts, but that is not by itself a reason for taking the distribution over initial conditions as the basic object.

As Albert points out, "once [a probability-distribution over possible microscopic initial conditions] is in place, all questions of what is and is not likely, all questions of what such probabilities to provide an interpretation of chance. One might wonder what is gained by defining chance in terms of another form of probability. The answer is that some very specific difficulties attach to the interpretation of chance, and defining chance in terms of Pr_G resolves these difficulties, while also explaining the epistemic role of chance and unifying the probabilities of quantum- and statistical-mechanical probabilities.

I. SINGLE-CASE AND GENERAL PROBABILITY

One of the most persistent sources of confusion in discussions of probability is the failure to distinguish general from single-case probability.[10] General probabilities apply to classes of events, and the basic form is conditional. The indefinite probability of *B* among *A*'s is written $\Pr(B/A)$.[11] Single-case probabilities, by contrast, apply to *particular* events rather than classes (that is, events that occur, or fail to occur, at a particular time and place), and the basic form is *un*conditional. Think of the difference between the general probability that an arbitrary roll of a pair of unbiased dice comes up double sixes and the single-case probability that *this particular roll* of *these particular dice* (which happen to be unbiased) on *this day* comes up double sixes. The former is conditional; it does not change over time; and there is an explicit reference class provided. The latter is unconditional; it has different values at different times; and there is no explicit reference class.

Single-case probabilities bear a direct connection to credence canonized in a principle that David Lewis dubbed the "Principal Principle."[12] General probabilities, by contrast, bear on credence only by way of their connection to single-case probabilities. The most important practical problem in assigning probabilities is that of determining which *general* probabilities to use to determine the single-case

was and was not to be expected, all questions of whether or not this or that particular collection of events happened merely 'at random' or 'for no particular reason' or 'as a matter of coincidence', are (in principle) settled," *op. cit.*, p. 10.

[10] General probabilities are also sometimes called "indefinite probabilities" and single-case probabilities called "definite probabilities."

[11] In logical terms, we can say that "Pr" is a variable-binding operator, binding the "*x*" in "$\Pr(Bx/Ax)$." I have suppressed the variables in the text.

[12] There has been dispute over the correct form of the principle, and various modifications have been proposed, one of which was accepted by Lewis before his death. I argued that the modifications are unmotivated in Ismael, "Raid! Dissolving the Big, Bad Bug," *Noûs*, XLII, 2 (June 2008): 292–307, but the disagreement need not detain us here. What matters for our purposes is something that is accepted by all parties, namely, that there is a tight link between chance and credence that any competent user of the concept knows at least tacitly.

probability of a given event.[13] If I want to know the single-case probability that the next roll of a pair of dice will come up double sixes, for example, there are indefinitely many, inequivalent general probabilities I might consult. There is the general probability that a roll comes up double sixes given that the dice are fair, or that a roll shows double sixes given that the immediately preceding roll was a double six, or that a roll shows double sixes given that I keep my fingers crossed.[14] Knowing which of these gives the right probability is highly nontrivial.

There are a number of views that one could have about the metaphysical relationship between single-case and general probability. One might hold that

(i) both are primitive forms of probability, neither reducible to the other,
(ii) general probabilities are definable in terms of single-case probabilities, or
(iii) single-case probabilities are definable in terms of general probabilities.

One of the difficulties in assessing accounts of probability is that some of them are offered as interpretations of single-case probabilities and others seem to be intended as interpretations of general probabilities, but they are compared as though they are competing accounts of the same object. This confusion runs deep in the literature, and there is little explicit clarity on the matter. Propensity interpretation and Humean reductions, for example, are seen most naturally as interpretations of single-case probabilities, whereas frequency interpretations are best understood as interpretations of general probability.

II. CHANCE[15]

By *chance*, I will mean the single-case probabilities that provide the link between the fundamental level of physical description in quantum mechanics and the measurement results that mark the points

[13] Cognoscenti will recognize this as the Reference Class Problem.

[14] One very striking example of the lack of clarity about the distinction between general and single-case probabilities is that the term "Born Probability" is used without disambiguation to refer both to the general conditional probabilities of the form $pr(\alpha/\Psi)$ derived from Born's Rule and to the single-case unconditional probability of α for a particular system in a state Ψ. Ask a physicist which one he means, and you are more likely than not to get a blank stare. It makes no difference when one is calculating because the general, conditional probability of (α/Ψ) and the single-case unconditional probability of some particular α in the future of a system in Ψ are quantitatively undistinguishable. But it makes a difference to interpretation. The two have a different logical form; they bear different relations to categorical facts; they treat past events differently; and so on.

[15] Eric Winsberg and Roman Frigg use "chance" more widely than I, to refer to any form of objective physical probability. See Winsberg, "Laws and Statistical Mechanics," *Philosophy of Science*, LXXI, 5 (December 2004): 707–18; and Frigg and Carl Hoefer, "Determinism and Chance from a Humean Perspective," in Dennis Dieks et al., eds., *The Present Situation in the Philosophy of Science* (New York: Springer, 2010), pp. 351–71.

of empirical contact between theory and world. In (standard, non-relativistic) quantum mechanics, states are represented by mathematical objects called "wave functions," and Born's Rule operates on wave functions to yield the chance that an observation or measurement on the system to which the wave function is assigned would yield a given result. Reiterated application of Born's rule generates a chance profile, that is, a probability assignment over results of possible measurements on the system. The chance of an event depends on the time at which it is assessed. Past events have a chance of 0 or 1; future events have chances in the interval between 0 and 1. More precisely, for any time *t* and any event *e*, we can assess the chance of *e* at *t*. If *e* is in *t's* past, it will have probability 0 or 1. If it is in *t's* future, it can take any real value in the interval between 0 and 1. An event that has a chance of 1 or 0 at *t* retains that value for all subsequent times. So, for example, if I hold a ticket to a lottery that will be decided by the result of a measurement carried out at noon on May 7, 2013, the chance of winning may have any value in the (closed) interval between 0 and 1 assessed at times leading up to that moment, whereupon it acquires a fixed value of 1 or 0.[16]

These quantitative facts—that past events all have value 1 or 0, that an event that has value 1 or 0 at one time retains that value at later times—are important because they are not things that we had to discover about the distribution of chances at our world. They are things that anyone who "grasps the concept" of chance will be able to tell you, and they are our best clues to its nature. Another clue to the nature of chance is provided by the aforementioned link to belief that Lewis identified and formulated in his Principal Principle (PP).[17] PP says that if we know what the chance of an event *e* is and we have no magical sources of information from the future, we should adopt the

[16] This leaves open whether chancy quantum events ever attain extremal values. It asserts only the conditional that *if* an event *e* occurs at a time *t*, ch(*e*) after *t* = 1.

[17] Lewis regarded the Principal Principle as the sole constraint, believing that it told us "all we know about the concept of chance." There is controversy about the correct form of the principle. The differences will not matter for our purposes, but for proposals for revision, see Ned Hall, "Correcting the Guide to Objective Chance," *Mind*, n. s., CIII, 412 (October 1994): 505–17; Michael Thau, "Undermining and Admissibility," *ibid.*, pp. 491–503; John T. Roberts, "Undermining Undermined: Why Humean Supervenience Never Needed to Be Debugged (Even If It's a Necessary Truth)," *Philosophy of Science*, LXVIII, 3 (September 2001): S98–S108; and Peter B. M. Vranas, "Who's Afraid of Undermining? Why the Principal Principle Might Not Contradict Humean Supervenience," *Erkenntnis*, LVII, 2 (September 2002): 151–74. A defense of the original version of PP can be found in Ismael, *The Situated Self* (New York: Oxford, 2007). The form adopted here is the one originally presented by Lewis, though he eventually adopted Hall's NP as a revision of the concept of chance. Lewis, "Humean Supervenience Debugged," *Mind*, n. s., CIII, 412 (October 1994): 473–90.

chance of e as our credence. If chances are to guide belief, we have to have ways of forming beliefs about chances. That means that we have to have some idea of what counts as evidence for statements about chance, and that evidence will have to be connected to the truth conditions for those statements. What plays that role is a connection to frequency. Getting that connection right requires paying attention to the distinction between general and single-case probability. Recall that general probabilities apply to classes and do not generally have a time index. Single-case probabilities, by contrast, are unconditional, time dependent, and pertain to particular occurrences. Chance is a single-case, time-dependent probability. $\mathrm{Ch}_t(e)$ is the chance that an event e has at t, where t is the time of assessment.[18] If e, the event that a coin toss to be made at noon on January 1, 2012 comes up heads, $\mathrm{Ch}_{\mathrm{now}}(e)$ is the present chance that the toss lands heads. In ordinary speech, we often suppress the temporal parameter, letting the conversational context decide the time at which the chance is assessed. If you ask me what chance I think you have of winning a bet predicated on the coin toss, for example, we both understand that you are asking for the chance that pertains at the time of asking. Now we are in a position to say how chance relates to frequency.

Bernoulli's law is a theorem that relates general probabilities to frequencies. It says that the relative frequency of *A's* in a typical ensemble of *B's* approaches $\Pr(A/B)$ as the size of the ensemble increases. This is good news; it gives us a necessary, probabilistic link between probabilities and frequencies. But there is a bad-news addendum: there are other theorems that tell us that that link cannot be strengthened. The possibility of divergence remains, no matter how large the ensemble.[19] This gives us the third constraint on the interpretation of chance, the chance-frequency link. The chance-frequency link says that if S is in state ψ, there is a necessary but approximate and probabilistic relation between the chance of observing e in an x-measurement on S and the relative frequency of e in x-measurements on systems in ψ.

We now have three clues to the nature of chance that we apply as constraints on interpretation;

(1) PP: one should set one's credence in a at t, equal to the chance of a at t, no matter what else one knows, provided one has no magical information from the future.[20]

[18] Not to be confused with the time at which e occurs.

[19] Thanks to Alan Hájek for prompting this clarification.

[20] Support for all of these constraints, which we can treat as partially, provisionally definitive of chance, is that someone who denied them would not fully understand the concept. Someone who knew that George Bush won the election in 2004, for

(2) Quantitative constraints: the chance of an event after it occurs is always 1 or 0; an event that has value 0 or 1 at one time retains that value for all subsequent times.
(3) Chance-frequency link: the relative frequency of *a's* in a typical ensemble of systems in ψ approaches the indefinite probability of a/ψ as the size of the ensemble increases, but the *possibility* of divergence in any finite ensemble remains, no matter how large the ensemble.

(1) supplies an analytic connection to credence. (2) and (3) concern the relationship between chances and categorical facts. Categorical facts are the nonprobabilistic, nonmodal facts about what actually occurs. (2) places restrictions on combinations of chances and categorical facts. (3) provides a necessary but ineliminably approximate and probabilistic connection between chance and frequency.

III. THE INTERPRETIVE DILEMMA[21]

The constraints are easily satisfied individually, but conjointly they present a dilemma that destabilizes both the standard reductive and nonreductive accounts of chance. The problem is that they specify connections between facts about chance and facts about categorical events which seem, on the one hand, too loose to permit reduction and, on the other, too tight to let us treat them as distinct existences. Why is the link between the chances and the categorical facts too loose to permit reduction? Bernoulli's law explicitly allows for the possibility that the chances may diverge arbitrarily far from the frequencies, which is a way of saying that it is a fact about the logic of chance that the very same distribution of actual events is *logically* compatible with an unlimited number of inequivalent chance distributions. The link between the actual pattern of events and the chances is irreducibly and irremediably probabilistic.[22]

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example, but denied that the present chance of his doing so is 1, or someone who knew what the chances were but did not think that they should guide his expectations, or someone who did not think that frequencies under the right conditions provided evidence for statements about chance, would not be regarded as understanding what chance is, or would be regarded as using the word in a different way.

[21] The interpretive dilemma distills a rather large body of literature to bare bones. For historical reasons, a good part of the philosophical discussion of chance is preoccupied with the compatibility of chance with Humean Supervenience (HS). HS is an independent metaphysical commitment, and it is not itself a constraint on the interpretation of chance. Lewis himself, the original defender of HS, held that the viability of HS hinged on the existence of an account of chance compatible with it, rather than the other way around.

[22] This raises questions about the Best-System Analysis of theories (BSA) discussed below, in connection with Loewer. The BSA purports to provide truth conditions for statements about probability, laws, and all of the apparently modal implications of a

Why is the link between the chances and the categorical facts too tight to let us treat them as distinct existences? There are necessary quantitative constraints on the relations between an event and the chance of its occurrence. Ask yourself whether *e* and the chances of *e* at different times can vary independently of one another. If these are distinct existences, there ought to be a possible world in which *e* occurs and the chance of *e* after the fact has any value we care to assign. So, for example, we ought to be able to conceive of a world in which a coin comes up heads and has a nonzero chance at later times of having come up tails. But the concept does not furnish a way of imagining the possibility. We can assign no intuitive content to the idea of such a world. This is a way of saying that certain combinations of categorical fact and values for chance do not make sense.[23]

This presents a dilemma for any account of the nature of chance. Chances have a peculiar, ontologically intermediate status that seems to frustrate both reduction to categorical facts and primitivism. From a formal point of view, chances look so much like ordinary physical quantities (they are represented by real-valued functions, they are assigned to particular systems, and they evolve in time) that it would be nice if we could treat chance as a primitive quantity, and say "so much the worse" for the Humean ban on necessary connections between distinct existences.[24] What is wrong with that reaction is that it fails to appreciate the regulative role that the Humean ban plays in these contexts. If we reject the Humean ban, we no longer have a way to recognize distinct existences. When we ask questions like "What is chance?" "What are colors?" or "What is goodness?" part of what we are trying to do is provide a compact, nonredundant catalogue of fact. In this context, necessary connections of a non-nomological nature

physical theory in terms of patterns in the manifold of categorical fact. It turns out, on this account, that to say that a statement *L* is a law is to say that the best overall systematization of actual fact treats *L* as a law. And to say that the chance that a certain photon passes a filter is .99 is to say that the best overall systematization of actual fact entails that the chance that the photon passes the filter is .99. The BSA recognizes, however, that statements of law and probability have a fundamental status in physics. It is best understood as a meta-scientific view.

[23] Again, the claim here is that you should have trouble assigning intuitive content to the possibility of a world in which past occurrences have a nonzero present objective chance of not having occurred. It is not that you cannot dream up a scenario in which we might choose to use the concept in a way that allows for that possibility. It is that as we presently use it, that possibility has not been *provided for*. I take it for granted that this is as close as we come to analytic truth.

[24] Accounts that identify chances with propensities, explicitly denying that propensities are grounded in intrinsic categorical properties, seem to be doing that.

function as a sign of redundancy. Here is a familiar pattern of argument: Tom asserts that *A*-properties are metaphysically distinct from *B*-properties. Alice responds that if *A*- and *B*-properties were distinct existences, there ought to be possible worlds in which the *B* facts are just as they are, but the *A* ones are wholly different, and *vice versa*. If there are no such possible worlds, Tom is wrong. The Humean ban on necessary connections between distinct existences functions in this pattern of argument as a test for ontological redundancy, and without it we no longer have a methodological foothold for addressing claims of metaphysical distinctness.

A rather large philosophical literature has built up around the interpretation of chance. Lewis's papers in the early 1980s brought the topic under special scrutiny because of the difficulty he saw with incorporating chance into his ontology.[25] There is a small collection of well worked-out interpretations. There are frequentist accounts, propensity accounts, primitivist accounts, nomic accounts, Humean accounts, and subjectivist accounts. Each of these, however, suffers from well-known problems. Each comes with one or another bullet to bite. Those working for reductions are responding to the fact that *e* and the chance of *e* do not behave like distinct existences. Primitivists are responding to the fact that reduction seems blocked by the ineliminably probabilistic character of the link between chances and frequencies. But neither can respond appropriately to pressures from the other side. As things stand, by general consensus, there is no entirely satisfactory interpretation of chance.[26]

IV. THE PROPOSED SOLUTION

There is a lesson that one learns in physics or math when one feels backed into a corner with a difficult problem: try the obvious solution. I want to suggest that there is a way out of these difficulties that is so obvious, once we have independent grounds for recognizing the existence of general probabilities, it can strike one as almost trivial. It takes a little unpacking to see exactly what is accomplished. The

[25] See Lewis, *op. cit.*

[26] There are many excellent critical discussions and surveys of the standard influential interpretations of chance. See Pollock, *op. cit.*; Michael Strevens, "Probability and Chance," in Donald M. Borchert, ed., *Encyclopedia of Philosophy*, 2nd ed. (Detroit: Thomson Gale/Macmillan Reference, 2006); and Craig Callendar, "The Emergence and Interpretation of Probability in Bohmian Mechanics," in Frigg and Stephan Hartmann, eds., "Probabilities in Quantum Mechanics," *Studies in the History and Philosophy of Modern Physics*, XXXVIII, 2 (June 2007): 351–70. Alan Hájek provides an especially nice general survey of interpretations of probability in "Interpretations of Probability," *The Stanford Encyclopedia of Philosophy*, ed. Edward Zalta (Spring 2010). URL: http://plato.stanford.edu/entries/probability-interpret/.

conceptual character of the quantitative constraints that an event's occurrence places on subsequent chances provided clear symptoms of nonbasicness, so reduction is needed, but not reduction to categorical facts. We reject both primitivism and reduction to categorical facts and define the chance of *e* assessed at *t*, written $\mathrm{Ch}_t(e)$, as follows:

Def: $\mathrm{Ch}_t(e) = \mathrm{Pr}_G(e/\text{pre-}t \text{ history})$

where $\mathrm{Pr}_G(A/B)$ is the general probability that a random pick from the *B's* will yield an *A*.[27]

It is easy to see that Def satisfies the second and third of the constraints on the interpretation of chance. It follows immediately from the definition that the chance of any event assessed after its occurrence will always be 0 or 1. And since Pr_G is a probability function, it is going to satisfy the probability axioms, and the chance-frequency link will hold. The nontrivial part of establishing that Def satisfies the constraints on interpretations is saying why the measure defined by Def should play the role characterized by PP in guiding belief. Recall that PP said that provided we have no magical information from the future, the degree of belief we assign to *e* should equal the chance of *e*. But there are any number of functions of Pr_G, any of which have the right form to play that role. Consider, for example,

$\mathrm{Pance}(e) = \mathrm{Pr}_G(e/\text{pre-2001 history of Australia})$
$\mathrm{Fance}_t(e) = \mathrm{Pr}_G(e/\text{post-}t \text{ history})$
$\mathrm{Trance}(e) = \mathrm{Pr}_G(e/\text{all of history})$

In contrast to these functions, what makes chance peculiarly suited for the role carved out by PP? The answer is that chance is the only one of these functions that trumps *all* and *only* historical information. For creatures like us, historical information is in principle available, whereas information from the future is out of bounds. Chance guides belief because it is probability conditioned on all information that is in principle available to a situated agent. As such, it is the only function of Pr_G that trumps all and only such information. Belief guided by Pance(e) and Fance(e) would lead us to assign probabilities other than 1 to historical events that we know occurred, or nonzero

[27] The definition assumes that the general-probability distribution ranges over all possible (historical) events, so that $P(E|H)$ is well defined for any possible world history *H*. So, strictly speaking, it applies in the first instance to theories that treat the world as a closed system, and hence to cosmological theories. It is in that context—a deterministic world—that it seemed probabilities were eliminable. In the case of open systems, probability appears explicitly in the form of a distribution over exogenous variables, and the status of that distribution depends on the context. In some cases, it is explicitly epistemic. In others, it is statistical.

probabilities to historical events we know did not occur. Belief guided by Trance(e) will not lead to those kinds of mistakes, but there is a different problem. Information about the "trances" of future events is in general beyond our ken and hence unavailable to guide present belief.

V. $\mathrm{PR}_G(A/B)$

Def does not provide a reduction of facts about chance to nonprobabilistic facts. It provides an interpretation of chance in terms of a form of general probability. The burden of argument then falls on questions about Pr_G. What is it? What reason do we have to believe in it? What sort of progress is made by reducing chance to $\mathrm{Pr}_G(A/B)$? Addressing these in turn: Pr_G defines the notion of randomness according to a particular theory. $\mathrm{Pr}_G(A/B)$ is the general, conditional probability that a random pick from some proscribed volume of phase space (B) will yield a system whose state—at that time or some later time—falls in another such volume (A).[28] Grounds for the recognition of Pr_G come from a number of sources. Loewer and Albert have argued for recognition of a form of objective probability to ground statistical-mechanical probabilities.[29] Sober has shown that evolutionary theory makes use of objective macro-probabilities and has given general arguments for the recognition of objective probabilities.[30] We do not need the examples of statistical mechanics, evolutionary theory, or special sciences to make the case. There can be little question that some measure is tacitly and routinely invoked as a matter of practical necessity in classical contexts with deterministic laws, because without such a distribution the laws are virtually impotent to govern expectations or generate testable predictions.[31] Global deterministic dynamical laws determine what is *possible*, but they do not tell us how to divide opinion among the possibilities. In any realistic situation, there are indefinitely many present states

[28] I remain agnostic on whether the backward probability $\mathrm{Pr}_G(A/B)$ where B occurs after A is essential. In an indeterministic setting, the backward probabilities may not be given. In a deterministic setting, they follow from the forward probabilities. See Guido Bacciagaluppi, "Probability, Arrow of Time and Decoherence," in Frigg and Hartmann, eds., *op. cit.*, pp. 439–56.

[29] See Loewer, *op. cit.*; and Albert, *op. cit.*

[30] See Sober, *op. cit.*

[31] This is a qualified conclusion. One can imagine laws that retain their predictive power even in the absence of precise knowledge, but the Newtonian dynamical laws do not. So the claim is that global determinism *by itself* places very weak constraints on the evolution of local subsystems of the universe. For examples of laws that do retain predictive power even in the absence of precise knowledge, consider laws that entail that any system takes the shortest path through phase space at a fixed speed to a final state S, no matter what its starting state and in a manner entirely immune to influence. To multiply examples, one just has to limit interaction and influence.

for the universe as a whole that are epistemically possible relative to our knowledge, and hence indefinitely many future states into which the universe can evolve.[32] The same is true for any proscribed subsystem of the universe because of the impossibility of perfectly isolating it from its environment. Any proscribed subsystem is in principle subject to innumerable possibilities of unanticipated interference. We can lower the probability of interference, but the possibilities are ineradicable. Laws that tell us only which future states are possible given our present knowledge will give an infinite, undifferentiated set of possibilities with no distinction between significant and negligible possibilities or between relevant and irrelevant influences.[33]

In practice, we deal with ignorance by randomizing over the values of unknown variables. Talk of randomness is not well defined without a choice of Pr_G. As we saw, Pr_G effectively *defines* what is meant by "random" and (hence) guides the assignment of probabilities where there is ignorance.[34] To see this, let W be a collection of physically possible worlds (or physically possible histories for the world), and let W^* be the set of worlds in which the system of interest s exists and possesses all of its known properties. If W^* constains more than one world, there will be indefinitely many probability distributions over W^*. When we talk about randomizing over the values of unknown variables, we are talking about choosing a distribution over W^*. It does not help to adopt an indifference principle; indifference principles can be applied consistently only over *equiprobable* alternatives. And it does not help to say that we should give a flat distribution, because there are different ways of partitioning any set of possibilities, and a

[32] The reference is to: "Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes." Pierre-Simon Laplace, *A Philosophical Essay on Probabilities*, trans. Frederick William Truscott and Frederick Lincoln Emory (New York: Cosimo, 2007), p. 4.

[33] One can, of course, deny that a deterministic theory generates expectation, holding that it tells us only what is possible and impossible. Everything else is done by the subjective probability distribution which one brings to a theory. In practice, that is not what we do. A canonical distribution is assumed, usually tacitly, that is not regarded as entirely discretionary. The explicit content of a theory needs to include a probabilistic postulate to lend the theory some testable content.

[34] A little more precisely, let S be a system of interest; let C be a summary of everything that is known about S; and let $A = \{A_1, A_s, A_3, \ldots\}$ be an unknown variable with values $\{A_1, A_s, A_3, \ldots\}$. Randomizing over A-values, holding C fixed, involves assigning each value of A the probability that a random pick from systems that satisfy C will yield that value. But that is just to say that it involves assigning the probability distribution over $\{A_1, A_s, A_3, \ldots\}$ generated by $\mathrm{Pr}_G(A_n/C)$.

flat distribution over one partition will yield a nonflat distribution over another. We could demand a flat distribution over the most fine-grained partition. This may deliver a univocal answer in the finite case, but it will not do so in the denumerably infinite or continuous cases. There is a long history of attempts to justify the choice of $Pr_G(A/B)$, and different views about whether and how the choice can be justified, but there is no way around the fact that some choice is needed. The point is prior to and independent of any particular account of the nature of chance.[35]

Choosing Pr_G is equivalent to choosing a standard that identifies partitions of W^* that divide it into equiprobable classes. In physical contexts, this standard gets embodied in the metrical structure of the phase space, and such strong intuitive prejudices operate in dividing possibilities into equiprobable classes that the choice of $Pr_G(A/B)$ often goes unmentioned and unnoticed. In classical mechanics, it did not come to light until disputes in the foundations of statistical mechanics bought it under scrutiny. Nowadays, as a result of those disputes, there is recognition among philosophers of physics that a form of statistical probability is both compatible with determinism and needed to derive a dynamics for larger-than-point-sized volumes of phase space. We can picture these higher-level dynamics as migration patterns which tell us how a class of systems distributed in a given way across a finite volume of phase space typically redistribute themselves over time.[36] To generalize the conclusion that the higher-level migration patterns are needed to derive expectations not only in the specialized context of statistical mechanics but in virtually any real-life calculation, we need only observe that the scientist working with any real-world system is always working with systems whose state can only be isolated, as an epistemic matter, in a finite volume of the universal phase space.[37] He appeals to the general information embodied in the large-scale migrations patterns to make up for his lack of specific information.

What lends $Pr_G(A/B)$ empirical content is the link to frequencies given by the Bernoulli principle. The bad-news addendum to the principle blocks a direct reduction to frequencies, but the probabilistic link is enough to make Pr_G empirically meaningful. I argued in

[35] This is a very quick summary of arguments from Ismael, "Probability in Deterministic Physics."

[36] To derive these higher-level migration patterns for any distribution, it suffices to say how a flat distribution evolves.

[37] Although it is still counter to orthodoxy, the idea that objective probability may be not just a quantum-mechanical phenomenon is gaining some currency.

The Situated Self that Pr_G should be treated alongside the physical laws as a primitive postulate whose correctness is justified by the success of the theoretical package of which it is a part. The success of the package as a whole confers whatever confirmation it possesses upon its elements. And confirmation is judged not by compatibility with the laws, but by the ability of the theory to predict the evidence with high probability. Note, however, that to say that Pr_G is a primitive theoretical postulate is to remain neutral on whether it can be reduced, eliminated, or analyzed as a particular form of idealized subjective probability at a meta-scientific level. It is possible for reasonable and well-educated philosophers to accept that laws of nature are physically fundamental but disagree about what laws are, whether facts about laws can be reduced to or analyzed in terms of some more fundamental set of facts, or whether we have good reason to believe that there are any laws. In the same way, it is possible for reasonable and well-educated philosophers to agree that a probability measure of the form $\mathrm{Pr}_G(A/B)$ is physically fundamental and disagree about what it is, whether facts about it can be reduced to or analyzed in terms of nonprobabilistic facts, and even whether we have good reason to believe in its existence.[38] Questions about which structures are *physically fundamental* are questions about the components needed for a working theoretical package. They can be treated independently of questions about the metaphysical nature or status of those components.[39]

But if Pr_G is accepted as a primitive postulate, the question of what progress is made by reducing chance to $\mathrm{Pr}_G(A/B)$ becomes pressing. The answer is that the definition of chance in terms of $\mathrm{Pr}_G(A/B)$ reduces it to a form of statistical probability that also underwrites the probabilities of statistical mechanics, explains the epistemic role

[38] This last depends on whether *x*'s being a fundamental postulate of science provides a reason to believe that *x* exists. Not all philosophers think it does.

[39] The recent debate between Maudlin and Loewer about the nature of laws is instructive in this regard. See Tim Maudlin, "Why Be Humean?" and "A Modest Proposal Concerning Laws, Counterfactuals, and Explanations," in *The Metaphysics within Physics* (New York: Oxford, 2010); and Loewer, *op. cit.* Both agree that laws are physically fundamental structures, as do, for example, David Armstrong, *What Is a Law of Nature?* (New York: Cambridge, 1983); and Bas van Fraassen, *The Scientific Image* (New York: Oxford, 1980). But they hold a wide range of views about the nature of laws. Maudlin denies that there is any physics-independent platform from which to raise questions about ontology, and regards physically fundamental structures as ontological bedrock. Loewer follows Lewis in thinking that laws (and everything else that exists) reduce to patterns in the Humean manifold of events. Armstrong holds that laws are relations between universals. One might just as well accept that they are primitive physical postulates and hold that they are convenient fictions, nodes in an uninterpreted calculus for deriving predictions about observable structures, or the real structures that underlie and explain the world's observable properties.

of chance, and, since $Pr_G(A/B)$ bears no necessary connection to categorical facts, resolves the dilemma generated by treating chance as fundamental. Once we clear up some hard-to-pinpoint confusions on the interpretation of chance, which stem from a failure to distinguish general from single-case probability, and appreciate the need for general probability in deterministic and indeterministic contexts, Def tells us that chance is nothing more than a form of single-case probability derived from Pr_G by conditionalization on the past. We can then focus our attention on the interpretation of Pr_G. By defining chance in terms of Pr_G, Def preserves the temporal symmetry of the fundamental theoretical postulates. It is also a nice feature of Def that Pr_G is not time dependent. There is a very real worry that because chances of past events always have values of 0 or 1, while chances of future events range in the (closed) interval between 0 and 1, admitting chance into physical theory recognizes a fundamental form of temporal asymmetry; it says that the past is in some fundamental respect different from the future.[40] It should be noted, however, that although Def reduces chance to a form of general probability, it is not the sort of reduction that Einstein envisaged. Einstein's hope was to reproduce the quantum-mechanical probabilities from deterministic laws and a probabilistic postulate on the model of statistical mechanics. The proposal here accepts the ineliminably probabilistic character of the dynamical laws but spreads the guilt by arguing that determinism offers no respite from probability. Even deterministic theories cannot make due in practical terms without a specification of Pr_G.

Why, and in what sense, is Pr_G objective? It is objective at least in the sense that it is not discretionary. I have said that is a form of objective probability. There is a lack of clarity in the literature about what the claim of objectivity amounts to. For me, it means that it is to be conceived as part of the content of a theory, defined independently of the subjective probabilities of its users.[41] Pr_G is implicated in prediction, confirmation, and explanation. These ought to flow from the theory rather than the judgment of the user. And the single-case probabilities derived from Pr_G under conditions of partial information *guide* rather than *describe* subjective probabilities. Winsberg and Frigg have

[40] This is a controversial topic. Both the characterization of temporal symmetry and the question of whether the existence of chance violates it are unsettled. See Bacciagaluppi, *op. cit.*

[41] This leaves a lot open. It allows, but does not require, an interpretation of theories as embodiments of expert opinion, so that chances turn out to be credences of an expert believer.

objected recently to what they regard as ontologically inflated interpretations of statistical-mechanical probabilities. They argue that the only stable position construes them as objectively constrained epistemic probabilities suited for agents with our particular epistemic profile. Pr_G is objective, but not ontologically inflated in their sense. I agree with their arguments, and chance (on my interpretation) is well suited to their view as a form of statistical probability poised to guide the opinions of agents with our particular mix of ignorance and knowledge.

VI. MAUDLIN AND LOEWER

This is a good point to relate what I have said to some recent work of Tim Maudlin and to say a little more in detail about Loewer. Maudlin's view is a form of what can be called "nomic probability."[42] In nomic accounts, the stochastic laws define a form of general, conditional probability, and the general probability is used to define chance by conditionalization on history. Maudlin writes:

> Let us take a deterministic FLOTE [fundamental law of temporal evolution] and adjunct principles that operate in a special relativistic space-time. Take a surface that cuts every maximal timelike trajectory in the space-time exactly once (a Cauchy surface)....Boundary values can be specified on this surface, such as the distribution of particles, intensities of fields, etc.....If the FLOTE is deterministic in both the past and future directions, then the boundary values will determine a unique distribution of the physical magnitudes through all time. Such a distribution describes a physically possible world relative to those laws.[43]

When we have deterministic laws and a specification of boundary values rich enough to define a Cauchy surface, the laws constrain history in the future direction to a single model and no probabilities are needed. But, he continues:

> If the FLOTE is stochastic....specific boundary values on the Cauchy surface yield not a single model but a set of models, corresponding to all of the outcomes permitted by the laws. Furthermore, the set of models consistent with some boundary values is invested with a metric over measurable subsets, a measure of how likely it is, given those boundary values, that the world will evolve into one of the members of that subset.[44]

[42] John Pollock coined the term. He was an early proponent of nomic probability.

[43] Maudlin, "A Modest Proposal Concerning Laws, Counterfactuals, and Explanations," p. 18. One can see the nod to Maudlin's paper in my title. Maudlin includes a footnote acknowledging qualifications about constraints and boundary conditions, but he leaves it at that.

[44] *Ibid.*, pp. 18–19.

The case that Maudlin does not discuss, and the one that brings the need for the fully general Pr_G into relief, is the deterministic case in which we do not have enough information to identify a unique Cauchy surface. If epistemic probabilities are to be empirically constrained, that is, if it is not an entirely discretionary matter what probabilities are assigned when we are dividing opinion among multiple epistemically possible models, we need an objective metric over measurable subsets of physically possible trajectories. This is so whether or not the fundamental laws of evolution are deterministic. In these cases, we need higher-level laws that describe what I referred to earlier as migration patterns over phase space, that is, laws that tell us not how systems in specific microstates are bound to evolve, but how systems whose states fall within some volume typically distribute themselves across that volume and redistribute themselves over time.[45] So, I agree with proponents of nomic probability that the notion of a stochastic law should be treated as more primitive than that of chance, and that chance (defined as "objective, single-case propensity"), should be defined as Maudlin says, effectively by conditionalizing on the past. But I disagree that probabilities are eliminable when we have deterministic laws. Nontrivial probabilities in deterministic contexts invoke the same kind of measure over physically possible trajectories that Maudlin recognizes in the indeterministic case, and they are implicated in every real-world derivation of expectation from theory.[46]

Loewer states his position in terms of a philosophical framework that I have reservations about, a development of Lewis's Best-System Analysis (BSA), which aims to reduce facts about laws, chances, and physical modality quite generally to facts about patterns in the manifold of categorical events.[47] But if we put aside the claim that the BSA provides truth, rather than, for example, *acceptance* conditions for a physical theory, our views are almost indistinguishable. There are some differences of detail, but we agree that probability is compatible with determinism, and that a probabilistic postulate included

[45] Note here that this does not depend on epistemic probabilities that are flat over the initial volume of phase space. Even if your epistemic state is represented by a distribution that is heavily clustered in one corner of the volume, for example, you need to know how ensembles clustered that way typically evolve.

[46] It is worth noting that Maudlin's definition of chance applies smoothly to the more general context…

[47] See Loewer, *op. cit.* My reservations stem from the fact that to be successful, the BSA has to recognize that even though the acceptance conditions of a theory can be given in terms that advert only to the pattern of actual events, statements about law and probability have modal implications that cannot be reduced to claims about actual events. In my view, the BSA has the wrong account of modality.

alongside the laws as a component of a theoretical package can be used to define chance.[48]

VII. WHY TREAT PR_G AS THE BASIC OBJECT?

I have said that what we need to derive expectations in practice—outside the context of pen-and-paper problems in which all of the information is artificially stipulated—are migration patterns over phase space which deliver the probability that a system which starts out in some finite volume ends up in another at some later time. These are not derivable in any context, deterministic or otherwise, without a probability measure put in somewhere. But one might ask: why treat $Pr_G(A/B)$ as the basic object? Why not some other form of probability? Why not, for example, a probability distribution over initial conditions? The first reason is that the idea of a probability distribution over initial conditions is notoriously problematic. What sense—empirical or conceptual—does it make to talk about a distribution over initial conditions if the universe occurs only once? $Pr_G(A/B)$, by contrast, gets empirical content from a probabilistic link to frequencies in finite actual ensembles, for example, the frequency of tosses of American quarters from 1976 that land heads, the frequency of vegetarians among French people, the frequency of smokers under 50 that get cancer.[49] These finite ensembles are the primary site of empirical application of probabilistic concepts. A distribution over initial conditions gets whatever empirical contents it has indirectly, from the constraints it places on them. Tracing all probability to a distribution over initial conditions of the universe obscures its empirical content, effectively reducing it to an empirically empty case.

The second reason for treating Pr_G as basic is that it cannot be generated from a distribution over initial conditions in indeterministic contexts. When dynamical evolution multiplies the number of possibilities, an initial distribution will not determine a final one. So it is a special feature of deterministic laws that they can generate a distribution at later times from a distribution over initial conditions. This is a good reason for thinking that Pr_G is the more general and basic form.[50]

[48] I would say something stronger, namely, that a probabilistic postulate is not just compatible with but practically indispensible for determinism, and that there is no measure of fitness with evidence without a probabilistic postulate.

[49] See Detlef Dürr et al.,"Bohmian Mechanics," in Borchert, ed., *op. cit.*; Dürr, Sheldon Goldstein, and Nino Zanghí, "Quantum Equilibrium and the Origin of Absolute Uncertainty," *Journal of Statistical Physics*, LXVII, 5/6 (1992): 843–907); and Callendar, *op. cit.*

[50] It is a special feature of bi-deterministic laws that they can generate a distribution at one time from a distribution at any other. The Newtonian laws are bi-deterministic.

VIII. NOVELTY

If the novelty introduced by indeterministic laws does not consist in the appearance of a new form of probability or even a new role for probabilities in inductive inference, wherein does it lie? It consists in the elimination of a degenerate case present in a deterministic setting. I have argued that probabilistic assumptions play an ineliminable role whenever we are working with volumes of phase space from which multiple physically possible trajectories emerge. In deterministic theories, the number of such trajectories goes to 1 as the size of the volume goes to 0. So eliminating historical ignorance reduces all prediction to the special case in which there is only one possibility. In indeterministic theories there are multiple trajectories through every point, so there is no degenerate case, and even if we eliminate historical ignorance a measure is needed to set credence. There is a way of putting this that makes contact with the technical work on hidden variables in quantum mechanics. In deterministic theories, even when we are working with imprecise or incomplete information, there is a more fine-grained description that locates the system in an underlying space with a single trajectory through every point. In the indeterministic case, there is no such underlying space, no fine-grained description with a single trajectory through every point that will let us reconstruct the probabilities at the higher level from a distribution over variables whose values are (in principle, jointly) measureable. We have to formulate the laws using a space that has multiple trajectories through some points, and that means that if we are to end up with an objective measure over possibilities, probabilities have to appear explicitly in the statement of the laws.

Can't we reinstate determinism trivially by "discerning" variables so hidden that they cannot be measured and use them to parameterize phase space with an intrinsic measure that generates the Born measure over quantum-mechanical state descriptions? This is a natural suggestion, and the one that Einstein explored until his death. Under such an arrangement, quantum-mechanical state descriptions would correspond to finite volumes of the underlying space in the same way that thermodynamic state descriptions correspond to finite volumes of the phase space of statistical mechanics. The problems with the suggestion are the topic of a vast and technically formidable literature exploring the possibilities for hidden-variable interpretations. There are unsettled questions, but we do know that any such interpretation will exhibit nonlocality or contextuality of some form at the fundamental level, and it is very difficult both in physical terms and conceptually to work out the resulting implications.[51]

All of this raises the question of why the interpretation of chance has operated under the dogma that chance is physically fundamental. It is not just that probabilistic assumptions work behind the scenes in deterministic contexts. Even where probabilities are in explicit use there is confusion about their status. I believe it is easy to slip into thinking that the probabilities represent facts about our epistemic states in part because they are only needed in the deterministic case when there is historical ignorance. But that is not right. What they represent is objective, statistical facts about migration patterns of typical ensembles in the general population, and we rely on these general facts when our specific knowledge gives out. Another reason, however, is a holdover from a conception of physical law built on the Newtonian example, according to which dynamical evolution is a species of causal *production*, so that dynamical laws tell us how one state of the universe produces the next in a manner that is temporally asymmetric and involves ideas of causal sufficiency.[52] In a relativistic setting, this conception of laws does not fit. Laws are reinterpreted as global constraints with no intrinsic temporal direction and no special connection with time. The distinction between law-like and contingent features of the world is central to any physical theory, but from the perspective of this new conception of law, it is an inessential and quite special feature of the deterministic case that contingencies can be isolated on a space-like hyperplane, so that once we fix the state of any such hyperplane the laws determine everything else. From this new perspective, there is no reason to expect the distinction between law-like and contingent features of the manifold to line up in this simple way with the division of space and time. In an indeterministic setting, contingency is distributed throughout space and time. From a relativistic point of view, this seems a more natural arrangement. It would seem a surprising accident if the world *did* turn out to be deterministic.

[51] This is so at least so long as space-time is retained as the fundamental arena of physical description. There are some interesting suggestions from multiple quarters that treat space-time as emergent and reinstate determinism and locality as the fundamental. For an excellent discussion of hidden variable results, see Carsten Held, "The Kochen-Specker Theorem," *The Stanford Encyclopedia of Philosophy*, ed. Edward Zalta (Winter 2008). URL: http://stanford.library.usyd.edu.au/entries/kochen-specker/.

[52] Maudlin, "A Modest Proposal Concerning Laws, Counterfactuals, and Explanations," does a very nice job of articulating this conception, which he himself endorses.

IX. METHODOLOGICAL REORIENTATION

I have been arguing that the philosophical literature has treated chance as a surprising and anomalous protrusion of probability into an otherwise probability-free environment, whereas in fact it is a special case of something more fundamental and perfectly pervasive, the tip of a probabilistic iceberg that is present but remains largely below the surface in deterministic settings. Accepting this prompts a methodological reorientation, shifting philosophical attention from chance to Pr_G. This shift of attention has implications for how we think of physical probability. First, unlike chance, Pr_G is not inherently dynamical. If we can talk about the probability of evolving from one subspace of phase space into another, we can talk equally about the degree of overlap between two subspaces (that is, the probability of being in *B*, given that one is in *A*). Second, Pr_G has no intrinsic temporal asymmetry. It defines nontrivial transition probabilities in both directions, from past to future and future to past. Third, we no longer have to suppose that there are two distinct types of physical probability: chance and the micro-canonical probability measure invoked by statistical mechanics. We have a single measure, Pr_G, from which both can be derived.[53] Finally, rejecting the dogma that chance is physically fundamental resolves the interpretive dilemma that chance suffers by focusing interpretive attention on a form of general statistical probability rather than single-case, dynamical probabilities. Statistical probability provides a much more intuitive entry point into the circle of probabilistic concepts.

X. APPLICATIONS

I have spoken nonrelativistically for intuitive ease, but I want to pause to say how to translate into a relativistic setting and mention a couple of applications. To translate into a relativistic setting, we identify systems with world lines, define chances at points, and substitute "contents of past light cone" for "history." So now we have:

Def*: $Ch_p(e) =_{def} PrG(e/\text{the contents of } p\text{'s past light cone})$

Traditional Laplacian definitions characterize chancy events as those that are undetermined by the dynamical laws from the preceding

[53] On Albert's formulation, classical statistical mechanics consists of three postulates: (1) the fundamental dynamical laws, (2) a uniform probability distribution—the micro-canonical distribution—over the possible phase points at the origin of the universe, and (3) a statement characterizing the origin of the universe as a special low-entropy condition. $Pr_G(A/B)$ is equivalent to the micro-canonical probability distribution. And if we put a low-entropy initial state in for *B*, it will give us the thermodynamic laws. See Albert, *op. cit.*

state of the universe. This leaves no room for chancy events in a universe governed by global deterministic laws, like the one proposed in various Everettian interpretations of quantum mechanics. One of the virtues of Def* is that it separates the existence of chance from determinism, tying it to the light-cone structure. If we replace the Laplacian definition with:

> The occurrence or nonoccurrence of an event is *chancy* just in case it cannot be predicted with certainty by application of the dynamical laws to the contents of (that is, to the open set of events that lie in) its past light cone,[54]

we preserve a connection between chance and predictability but break the connection with determinism, because the connection between predictability and determinism is lost in the context of a light-cone structure that imposes greater restrictions on the availability of information. The result is that if we apply the definition in an Everett universe, we get chancy events despite the determinism of the global dynamical laws. To see how this works, suppose that I am an observer in an Everettian universe about to carry out a spin measurement on a spin half particle in a single state. Suppose that there is actual splitting into separate, spatiotemporally disconnected branches of the universe and that there is no evolution between the time of the measurement and observation of a result. There will be two actual observations by downstream descendents of myself. One (let us call her "Jenann-up") will observe spin-up. The other ("Jenann-down") observes spin-down. Both observations will be chancy according to the definition because neither Jenann-up nor Jenann-down will be able derive the content of her particular observation with certainty from nomic probabilities by conditionalizing on the (pre-observation) contents of her past light cone. The best that she will be able to do is assign probabilities in accord with the typicality of those results in physically possible trajectories that match the past light cones of the observation event. And we can give quantitative significance to the Born probabilities. They function just like the general probabilities in a classical setting, as derived from a metric over measurable subsets of physically possible trajectories obtained by conditionalizing on the contents of the back light cone.[55] The probability of observing α in an O-measurement on a system in state Y is $\mathrm{Pr}(\alpha/O\&\Psi)$.

Another nice application of specialized interest is that Pr_G gives us a natural interpretation of the notion of typicality that plays a central

[54] For ease of definitions, I include e in its future light cone but not its past.

[55] In this case, all physically possible trajectories compatible with a pre-measurement history are actual, but they work just as in Maudlin's account of the indeterministic case.

role in the Dürr, Goldstein, and Zanghi (DGZ) version of Bohmian Mechanics.[56] DGZ explicitly disavow a construal of typicality in terms of probability on the grounds that it is empirically empty to talk about the probability of initial conditions. As Craig Callendar writes:

> An instinctive negative reaction to assigning probabilities to initial conditions is natural and probably even healthy. DGZ are understandably reluctant to dub the universe probable, for it invites quasi-theological pictures of supernatural beings picking the universe out of an urn.[57]

As mentioned above in connection with reasons for taking $\mathrm{Pr_G}$ rather than a distribution over initial conditions as basic, $\mathrm{Pr_G}$ gets its empirical content from the constraints it places on observed statistics in smaller than universe-sized samples.

XI. ADDRESSING OBJECTIONS

A couple of objections are worth addressing before closing. The first objection is that Def tells us that the chance of *e* at *p* is the single-case probability obtained from $\mathrm{Pr_G}$ by conditionalizing on historical information accessible from *p*. In fact, nobody ever possesses full historical information, so chances are epistemically inaccessible in practice. So how can they guide belief in the manner dictated by PP? The answer is one that I have defended in more detail elsewhere.[58] I hold that PP correctly captures the conceptual connection between chance and belief, but in practice what applies is a generalization of PP that accounts for historically based ignorance by forming a mixture of chance functions obtained by conditionalizing on epistemically possible histories whose weights reflect our credence that those histories are actual. In short, when we do not know what the chances are, belief is guided by our best estimate of the chances.

An interesting addendum to this response arises from a reply by Alan Hájek. Hájek observed that the response I just gave threatens to undermine my answer to the earlier question of why $\mathrm{Ch}_t(e)$ is peculiarly suited to play the role carved out by PP in guiding belief. Consider in particular the trances which are obtained from $\mathrm{Pr_G}$ by conditionalizing on all of history: $\mathrm{Trance}(e) = \mathrm{Pr_G}(e/\text{all of history})$. The reason for denying that these guide belief in the manner dictated by PP was that if we do not have crystal balls, we do not in general know what the trances are. But once we allow that in practice we do not know what the chances are either, the door is opened for

[56] Dürr, Goldstein, and Zanghí, "Quantum Equilibrium and the Origin of Absolute Uncertainty."

[57] Callendar, *op. cit.*, p. 367.

[58] See Ismael, "Raid! Dissolving the Big, Bad Bug."

arguing that the trances do as good a job guiding belief as the chances.[59] After all, Trance(e) agrees with $\text{Ch}_t(e)$ on past events, and it does at least as well as the chances on future events. And it is certainly true that *if* we know the trances we ought to adopt them as our credences, no matter what other information we have, since Trance(e) assigns probability 1 to all and only truths. If the only objection to adopting the trances as credence is that they are epistemically inaccessible (indeed, we are precisely as ignorant of the trances as of the future), we can accommodate ignorance of the trances in the same way we do with the chances by forming a mixture of trances obtained by conditionalizing on epistemically possible futures.

That is correct, so I need to say something a little more complicated than my earlier response. Notice that provided we use the same weights over epistemically possible futures in forming our best estimate of the trances and chances, our best estimate of future trances will be quantitatively indistinguishable from our best estimate of the chances. To say that our best estimate of the chances will be indistinguishable from our best estimate of the trances, however, is in its turn to say that in absence of information from the future, our best estimate of the chances is also our best estimate of the truth, since Trance(e) assigns probability 1 to all and only truths. So you can use the trances to guide credence if you want, but the calculation is a little more difficult. The chances and the trances come apart only in the presence of crystal balls.

This reinforces our explanation of the epistemic role of chance and tells us something interesting about the connection between chance and truth. Recall that PP said that provided that we have no magical information from the future, we should adopt the chances as our credences. We see now that we can derive a slightly more precise version of PP from the analytic and unqualified claim that truth should guide belief by adding a substantive, general claim about our epistemic position.

- If you know the truth, no matter what other information you have, adopt it as credence.
- The only specific information a situated human agent has about the world is information about past events.
- If you know the chances, provided you have no magical information from the future, adopt them as credence.

The lesson here is that chance is obtained from truth by forming a mixture over epistemically possible futures.

[59] The arguments that Pance$(e) = \text{Pr}_{\text{G}}(e/\text{pre-2001 history of Australia})$ and $\text{Fance}_t(e) = \text{Pr}_{\text{G}}(e/\text{post-}t\text{ history})$ do not guide belief remain in place. Pance(e) would have us assigning crazy credences, and Fance(e) would have us assigning credences other than 1 (or 0) to events we know have occurred (or have failed to occur).

This also sheds some light on the question of what makes general probabilities conditional on world history more epistemically relevant than general probabilities conditional on background knowledge. Or perhaps better—why, if we have the latter, should we care about the former? The former are not more important than the latter, but they are prior in the sense that they provide materials used in calculations aimed at obtaining the latter. Many of the quantities we keep track of in physics have this character. They contain general, or generalizable, information that can be combined with locally available information to yield solutions to problems encountered in context. We can think of them as partially prepared solutions that get tailored for application by a relatively simple procedure. This strategy of partially preparing solutions is something we use every day to simplify frequently encountered problems. Look in your fridge for examples of partially assembled solutions to the problem of making dinner. Look at an electrician's toolbox for partially prepared solutions to the problems he encounters in his work. Or look in a mathematics textbook for useful theorems that play the same role for math students.

The second objection charges that we have reduced one form of probability to another but have made no progress toward explicit reduction of probabilistic facts to nonprobabilistic ones. To this, I reply as before that physics can, and in my view should, leave Pr_G unreduced, treating it on a par with other physical modalities. If one is after reduction, the prospects are much better following the Best-System Analysis in taking the whole theoretical package as the unit of reduction. But there are many (myself included) who deny that reduction is called for.

XII. CONCLUSION

Physics is and has always been awash with probabilities. Probabilities are not derivable from laws that constrain only possibility either in the deterministic or indeterministic case, and they are indispensible for purposes of prediction and confirmation in both. Physical laws that constrain only possibility do very little of the predictive work in practice. The same goes for confirmation. The existence of evidence that is logically incompatible with a theory is (almost) unheard of. All confirmation goes by way of probabilities; data disconfirms a hypothesis not by rendering it impossible, but rendering it unlikely, and is only as objective as the probabilities invoked in comparisons of likelihood.[60] Something similar is true for explanation. There are always indefinitely many hypotheses that might explain a phenomenon. A

[60] There are complex issues about how likelihood figures in confirmation, many of them unsettled, but there is no question that likelihood comparisons are indispensible in hypothesis testing.

good explanation is one that makes the phenomenon in question not physically necessary or inevitable, but *probable*. Once we recognize a form of general probability that is present in deterministic and indeterministic contexts alike, chance can be regarded as a form of single-case probability with a special epistemic role derived by conditionalizing on the past. This resolves the interpretive dilemma that treating chance as a fundamental form of probability generates, unifies classical and quantum probabilities, allows us to explain the role chance plays in guiding belief, and shows us how to see chance not as a new and fundamentally different form of probability but one rooted in a more familiar form of statistical probability.

There are some general lessons to be learned from this example. Chance, on my view, is an intermediate quantity, defined in terms of the fundamental ontology in a manner designed to play the role carved out by PP in guiding belief. It has an objective extension (if statistical probabilities are part of the mind-independent fabric of reality, then chances are too), but the special connection to credence is captured in PP. PP is not a binding principle of reason, something that holds for all rational creatures under all circumstances, but a principle of epistemic rationality that holds for creatures like *us*, for the *most* part, operating under quite specific constraints and facing quite specific tasks. We can imagine creatures faced with different tasks and operating under different constraints, or extraordinary situations into which we ourselves might be thrown in which it would not make sense to adjust belief to chance. Chances would be equally present under those conditions, but they would not play the same role in guiding belief.

Chance is one example of a class of scientifically important structures that have this intermediate status. The interpretive problem for such quantities is to secure internal connections to perception and action while possessing objective truth conditions. Loewer, for example, characterizes the problem faced by interpretations of chance thus: "how could anything objective rationally constrain degrees of belief?" How could anything *other than* p *itself* that is part of the mind-independent fabric of reality rationally constrain degrees of belief in *p*? He argues that most influential accounts either fail to make chance objective or fail to secure the connection to belief.[61]

[61] "One reaction to the problem of providing a rationale for the PP is to give up on the idea that chance is an objective feature of reality and try to make do with just degrees of belief….Although this approach has been developed with ingenuity I think it is not likely to succeed. For how can this view account for the chances that occur in a fundamental theory like GRW? It seems it must treat GRW as a compilation of recommendations of the degrees of belief one should have given various situations. But why should one accept these recommendations as opposed to others? If the answer

Weslake and Price characterize the problem for accounts of the temporal arrow of causation and counterfactual dependence in similar terms. They say that such an account should not only identify the arrow with an objective feature of the world, but also explain its connection to action.[62] And like Loewer, they argue that influential accounts fall on one or the other horn of this dilemma.

Here, the interpretive strategy was to let the internal role of chance rigidly fix its extension, characterize that extension in objective terms, and then give a substantive account that adverts to contingencies of our epistemic and practical situation that explain why *those things* should play *that role*. I think this strategy can be employed usefully in other cases. I would argue that causal structure and other quantities with intrinsic temporal asymmetries, for example, can be understood in these terms.[63] The general viewpoint here is a pragmatic reorientation that recognizes that physical theories are practical and epistemic tools. They do not just serve as representations. They help us form beliefs about parts of space-time we do not have direct, independent sources of information about, and they help us intervene in nature effectively. They are not part of the fundamental ontology. They are best thought of as partially prepared solutions to frequently encountered problems. They are highlighted, brought into relief, and appear explicitly in formulations of our theories not because they have a distinguished ontological status, but because they play a special role in the practical and epistemic lives of creatures like us.

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is that GRW's recommendations are, in some sense "better" then the question *what features of objective reality make them better?* If there are such features that *they* will be the truth makers of chance statements." Loewer, "David Lewis's Humean Theory of Objective Chance," *Philosophy of Science*, LXXI, 5 (December 2004): 1115–25.

[62] This is their practical relevance constraint. In their words, any adequate account should "help us to make sense of a matter of great practical importance in our lives, the fact that we can act for future ends but not past ends (at least in normal circumstances)." Huw Price and Brad Weslake, "The Time-Asymmetry of Causation," in Helen Beebee, Christopher Hitchcock, and Peter Menzies, eds., *Oxford Handbook of Causation* (New York: Oxford, 2009), pp. 414–43.

[63] Price has argued something similar; see his "Causal Perspectivalism," in Price and Richard Corry, eds., *Causation, Physics, and the Constitution of Reality: Russell's Republic Revisited* (New York: Oxford, 2007), pp. 250–92. It is his view, however, that these quantities are perspectival. That is not my view. Perspectival quantities are characterized by variation of truth conditions with contingent facts about their users or the epistemic or practical context in which they are employed. On my view, the truth conditions of the quantities in question are fixed and independent of facts about users or context of use. Discussion of these comes at the pragmatic level, in explanations of the role played by those quantities in our epistemic and practical lives.