

On Chance (or, Why I am only a half-Humean)ⁱ

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Before the development of quantum mechanics, most of the philosophical discussion of probability focused on statistical probabilities. Philosophers of science have a particular interest in statistical probabilities because they play an important role in the testing and confirmation of theories, and they played a central role in the statistical mechanics of Boltzmann and Gibbs developed in the 18th century. Since the introduction of quantum mechanics, however, much of the philosophical attention has become focused on the interpretation of chances. These are the probabilities assigned to particular events (the detection of a photon at a certain location on a photographic plate, or the registration of the result of a spin experiment on a particular electron) by applications of the Born Rule. The appearance of chances in quantum mechanics marked the first time that probabilities made an explicit appearance in a *fundamental* theory. They raise new kinds of ontological questions. Unlike statistical probabilities (which pertain to classes of events), chances are single-case probabilities. And unlike credences (which represent the epistemic states of believers), chances purport to represent features of the physical world.

Hall's paper (this volume) introduces the main divide in the philosophical discussion of chances, and shows how the difference in orientation between Humean and anti-Humean views shapes the detailed development of those views. In this paper, I defend a half-Humean view that retains the ontological thesis that motivates Humeanism, but denies that the Humean account does (or should) provide a content-preserving reduction of statements about chance to statements about non-chancy facts. The first part of the paper is expository. In sections I and II, I sketch the history of Humean accounts of chance. In section III, I introduce the form of the contemporary Humean account. In section IV, I introduce the two arguments against Humeanism that (in my assessment) cut closest to its philosophical heart. The second part of the paper makes a positive contribution to the development of the Humean account. In section V, I situate chances in the matrix of probabilistic notions. In section VI, I say why I think Humeans should not try to reduce chances to non-chancy facts, and in section VII, I introduce the *half*-Humean view that I favor. This is an entry into a lively and ongoing discussion.ⁱⁱ No attempt is made to provide a comprehensive survey of arguments for and against Humeanism. The reader is encouraged to look into some of the discussions indicated in the endnotes.

I History

Questions about the nature of chance were part of the general ferment surrounding the interpretation of quantum mechanics in the foundations of physics for most of the 20th century, but David Lewis propelled them to the forefront of metaphysician's attention with a 1980 paper (Lewis, 1980). One of the central questions in the metaphysics of science has always concerned the status of modal notions: laws, causes, dispositions and capacities. At the time that Lewis was writing, the debate about these notions had settled

into two broad classes of view: so-called Humean and anti-Humean views. The Humean holds that the world consists of what happens: just one thing and then another, arranged in a four-dimensional manifold of events, the totality of local matters of particular fact. According to the Humean, laws and chances are patterns in the manifold of events. For the anti-Humean, they *govern* and *explain* those patterns.ⁱⁱⁱ In Lewis' eyes, the program of Humean metaphysics hinged on the possibility of providing a Humean reduction of chances. He had developed a powerful framework for articulating the difference between the Humean and anti-Humean position, and provided successful Humean reductions (by his lights and the lights of many of his followers) of laws and causation, but he despaired of providing a Humean reduction of chances. The result of this was that questions about the nature of chances aligned with perhaps the central dispute in the metaphysics of science, and it became a lightning rod for debate.

II The big bad bug

Here is how Lewis framed the issue. He assumed nothing about what the chances are quantitatively. In his mind, it was the purview of physics to tell us what the values the chances take at different points in space and time, just as it was the purview of physics to tell us the values of the electromagnetic fields. But it was the purview of metaphysics to try to understand what sorts of things chance *are*. Are chances objective features of the physical world? Do they supervene on non-modal facts, or are they fundamental features in their own right. How do they fit into the catalogue of Being? He introduced a principle thought that a connection to belief provided everything that we know pre-theoretically about chance. He called this the Principal Principle (PP). The task for the Humean, as he saw it, was to find something that supervenes on the collection of local matters of fact, which could play the role of chance guiding belief expressed by PP. The problem was that he thought it couldn't be done. The reason was laid out in a quite lovely argument in his 1980 paper. In the time since its publication, a massive literature has built up around the various ways of addressing Lewis' worry. A number of solutions have been proposed, including one that Lewis himself accepted (Lewis 1994).^{iv} And, since the issue touches on so many debates in the metaphysics of science, those discussions have been quite generally sharpened and deepened.

III The canonical form of the Humean view; the Best Systems Analysis

The philosophical motivation for the Humean view, as noted by Hall, is a metaphysical vision of the world, combined with a view about the epistemic role that beliefs about chance play. David Albert, one of the most influential contemporary defenders of Humeanism, puts it thus:

“[On the Humean view] the world, considered as a whole, is merely, purely, there. It isn't the sort of thing that is susceptible of being explained or accounted for or traced back to something else. There isn't anything that it obeys. There is nothing to talk about over and above the totality of concrete particular facts. And science is in the business of producing the most compact and informative possible summary of that totality.” (Albert, 2015, p. 23-4)

The canonical form of the Humean view is given by (what has come to be known as) the Best Systems Analysis. We are told that beliefs about laws and chances come in packages (“best systems”) that are chosen on the basis of simplicity, strength, and best overall fit with the Humean mosaic. Beliefs about laws and chances are products of the systematization of information about the Humean Mosaic. The patterns in the Humean mosaic that provide the basis for choice between different systems are then presented as truth-makers for chance and law assertions. On this view statements about chance are compact summaries of information about distributed patterns in the manifold of events. The function of this kind of compact summary is to provide limited creatures information that will guide action and belief in a world too complex to be fully comprehended in a description we could grasp.

There is a lot of room under the Humean umbrella for different accounts of what makes a system a good one, and whether there is a single system for all of science, or many systems, one for each special science.^v The Humean project can be thought of as a schema to be completed by providing an explicit account of how systematizations are chosen and individuated.^{vi}

There are open issues in the development of the program. The most important of these is that nobody has given an adequate, explicit account of what simplicity and strength are. Lewis himself tied the measure of simplicity and strength to his account of natural predicates, a part of his metaphysics that most contemporary Humeans among philosophers of science would rather do without.^{vii} It remains an outstanding task for the Humean account to fill this hole.^{viii} With that said, the Humean view has many proponents in the philosophy of science. It does a good job of capturing the function of scientific theories: viz., to systematize information about the Humean mosaic in a compact form for use by limited agents. It achieves a good match between the *function* of a scientific theory and the standards by which scientific theories are judged. Simplicity, strength and fit make good sense as standards by which theories are judged against one another, if the goal is to systematize information about the Humean mosaic in a compact form. And it doesn’t come with metaphysical commitments that seem at odds with an empiricist orientation. In sum, the Humean offers an account of how (chance+law) packages are formed that is meant to reproduce the epistemology of science, and she asserts that there is nothing more to being a law than being a theorem of the best system, and nothing more to being the correct distribution of chances than being the distribution entailed by the best system. The result is a metaphysically conservative view that holds that the fundamental modal postulates of a theory - the laws and the chance distribution - are nothing more than compact statements that encode information about the actual pattern of events. There are some outstanding questions that need to be answered in the development of the Humean account, but it remains a live program with many adherents.

IV Two arguments against Humeanism

There are two quite powerful objections against Humeanism. The first one alleges that the Humean account robs the laws and chances of explanatory power. It holds that chances are a substantial ontological posit needed in the explanation of why there are stabilized relative frequencies. The argument here parallels the

argument for believing in anti-Humean laws. The claim is that without laws as substantial ontological posits, there is no explanation for all of the regularity in the world. What keeps the planets in orbit, and airplanes from falling out of the sky? If not the laws, then what? What keeps casinos in business and insurance companies making money? If not the chances, then what? Laws and chances, according to the anti-Humean, govern events and explain regularities. She will point to instances of scientific explanation that invoke laws and chances in an explanatory role.^{ix} She will bolster her case by urging that we have no better guide to metaphysics than physics. If laws and chances are primitive elements in our physical theories, we should treat them as a primitive part of our ontology.^x What reason could we have for seeking reductions? She goes on to point out that in the internal logic of a theory laws and chances are invoked in explanations of phenomena (Emery 2015). In sum, the anti-Humean holds that Humean chances can't play the explanatory role that chances play. She further questions the Humean methodology asking what warrant there could be for a *meta*-physical viewpoint for reducing quantities that are treated as primitive within our best physical theories. Where the Humean says that laws and chances describe patterns in the manifold of fact, the anti-Humean says that laws and chances are independent existences that *govern* and *explain* the pattern of fact. According to her, theorizing is all aimed at identifying the explanatory substructure behind the facts.

The second anti-Humean argument contends that the BSA is the development of an old tradition that tries to reduce probabilities to frequencies, and holds that it suffers from the same problem that the simpler accounts in that tradition suffer from. The frequency theorist says that probabilities are to be identified with frequencies that satisfy some criterion C (let's call these the C-frequencies), and the anti-reductionist points out that the law of large numbers assigns a non-zero probability to the possibility that the C-frequencies diverge from the probabilities. That seems to be true no matter what one fills in for C, and presents an obstacle to identifying beliefs about chances with beliefs about frequencies of any kind.^{xi} The holistic reduction of (chance + law) *packages* to patterns in the manifold of actual events makes it hard to apply the law of large numbers directly, but it still seems to suffer from a version of this problem. If we look at the modal implications of accepting a system B as the Best System, we will find ourselves committed to the possibility of worlds in which the laws and chances aren't given by the best system *at that world*. So, for example, consider a simple world w, which consists of a sequence of a very large sequence of flips roughly half of which come up head and half of which come up tails, in a pattern that doesn't admit of any compression. And suppose that the best systematization (B) of the results of the flips at w assigns 50% chance to heads on any given toss. Now, B has a model in which every toss comes up heads. The best systematization of the facts of that world would assign a 100% chance to heads. From the point of view of B, that is a lucky accident, but it is one whose possibility is explicitly recognized by B. It is hard not to speculate that any Best System (i.e., any package of laws and chances that systematizes the facts at a world of complexity close to ours) is going to have models in which nothing of much interest happens, and which permits of a simpler systematization. So just as the very logic of probabilistic belief, as expressed by the law of large numbers, explicitly recognizes an ineliminable modal gap between probabilities and frequencies (of any kind that can be explicitly characterized by a criterion C). So, it seems that the logic of Best Systems

makes room for the possibility of worlds in which the laws and chances are not given by the best system (at that world).

The first of these arguments is an expression of what Hall calls the difference in orientation between Humean and anti-Humean view. It is question begging as an argument against Humeanism, because the Humean simply rejects the explanatory demand. The second argument is not easily dismissed. It is one of a family of arguments that purport to show on non-partisan grounds that the Humean account of chance fails to provide a reduction of chances to non-chancy facts, by showing that the truth conditions for statements about chance have a modal component that can't be paraphrased by any set of statements about purely categorical facts.^{xii} I think that the conclusion of these arguments is correct. In the remainder of the paper, I want to propose a direction in which the Humean view should be developed.^{xiii} The suggestion will be that we should accept the modal commitments of accepting a Best System, but give an account of them that understands them purely in terms of their epistemic role.

V The matrix of probabilistic notions

To pave the way for that, there is another adjustment to be made to the Humean program. The strategy for Humeans and anti-Humeans alike has been, for the most part, to attack the question of what chances are directly. Since chances appear along-side laws in our fundamental theories, the presumption has been that they are the most fundamental form of objective probability. The task has been conceived as a matter of sorting out the connection between the chances and categorical facts directly, while imposing the connection to credence captured in PP (or one of its proposed variants) as a constraint. If we situate chances in the more complex matrix of probabilistic notions, however, we get a more nuanced understanding of how chances relate to the categorical facts, and that paves the way for the Half-Humean view I want to propose.^{xiv} That is what I am going to do in this section. The discussion will be condensed, but details can be found in references.

A probability measure is a function, $P[\cdot]$, that maps events in the sample space, S , to real numbers such that:

$$P[A] \geq 0 \text{ for any event } A.$$

$$P[S] = 1.$$

$$P[A_1 \cup A_2 \cup \dots] = P[A_1] + P[A_2] + \dots \text{ for any countable collection of mutually exclusive events } A_1, A_2, \dots$$

Conditional probability of the event A given the occurrence of the event B is $P[A|B] = P[AB] / P[B]$ (AB : short for $A \cap B$)

We distinguish general probabilities from single-case probabilities. General probabilities pertain to classes and the basic form is conditional. Single-case probabilities (e.g., the probability that a particular carrier of the BRCA2 gene will develop cancer) pertain to individual events and the basic form is unconditional. General probabilities are related to single case probabilities by the principle that the single case probability of an x

that is randomly selected from the population of y 's is the general probability of x , given y . These definitions license an inference from a general probability to a single case probability where the selection procedure is random, and where nothing else is known that might affect the probability. So, for example, if the general probability that a person will develop cancer given that they carry the BRCA2 gene is 0.45, then the single case probability that a particular carrier of that gene will develop cancer is (*ceteris paribus*) 0.45.

Statistical probabilities are general probabilities. What makes them *probabilities* is that they obey the probability axioms. What makes them statistical is a connection to statistics. The connection to statistics comes from the law of large numbers (in its weak or strong version) which says that the relative frequency of x in a class of randomly selected y 's approaches the general probability of x , given y , with increasing probability, as the class gets larger.

In operational terms, statistical probabilities are elements in a theoretical matrix that mediate inference from observed statistics in local samples to others in the same class. Probabilistic notions are connected to actual observed frequencies by the body of operational procedures and norms by which we infer probabilities from collected statistics. The operational procedures are embodied in the canons of statistical inference.^{xv}

Assignment of statistical probabilities commits one to the expectation that the statistics for one sample will reflect those of others (provided that no selection was exercised either in the collection of the sample or in the target class, i.e., the one that we are forming expectations about.^{xvi} The expectation grows as the size of the sample increases, and is defeasible by the belief that the selection was biased, or that the selection process was not random.

Statistical probabilities contrast with epistemic probabilities. Where statistical probabilities are interpreted by a connection to frequencies, epistemic probabilities are interpreted by a connection to belief. What makes them probabilities is (again) that they obey the probability axioms. What makes them epistemic is a connection to belief. There is a descriptive form of epistemic probabilities. These are the credences. They represent the epistemic states (or degrees of belief) of agents.

The Principal Principle (PP) was introduced by Lewis as an expression of the role that chance plays guiding belief. His informal statement of the connection to belief was that if you know that the chance of e is x , and you don't have a crystal ball, or any other form of supernatural information from the future, your credence in e should be x . Few would nowadays agree with Lewis that PP captures 'all we know about chance', but almost everyone agrees that PP (or something quite close to it) captures a connection between chance and credence that acts as a constraint on the interpretation of chance.^{xvii} For that reason, the principle continues to play a central role in the philosophical discussion of the metaphysics of chance. PP tells us how chance is situated in this matrix. It tells us chances are a normative form of epistemic probability. And it tells us that the chances are adopted as credences in the absence of information from the future.^{xviii}

Now the question is how do we connect this matrix to the categorical facts. We begin with the most direct point of contact; viz., the general statistical probabilities associated with stabilized relative frequencies across reference classes of the kind that casinos, lotteries, and insurance companies rely on.^{xix} These kinds of probabilities are defined for types rather than tokens, and the basic form is conditional. They exist in deterministic as well as indeterministic contexts. We have a probability for a/b when we have a relative frequency of a 's among b 's with the right kind of stability (i.e., where the frequency is roughly stable across not carefully chosen subselections from the b 's).^{xx} We have good philosophical models of these kinds of probabilities in specific cases (the best known is the Diachonis model of the coin flip). The dynamical underpinnings, however, will vary from case to case, and the knowledge that there is a probability associated with an event in some reference class (i.e., a relative frequency with the right kind of microstructure) often precedes our explicit understanding of the dynamical underpinnings. Science was neck deep in these kinds of probabilities long before quantum mechanics came on the scene, and probabilistic thinking in everyday life relies on the existence of these kinds of emergent stabilized relative frequencies that let us form reasonable expectations where we have only partial knowledge.^{xxi}

We can sketch a broadly Humean story about the emergence and use of probabilistic thinking. Humeans think that there are physical systems and their categorical properties. They think that there are laws that determine the physically allowable trajectories. So far, there is nothing 'probability-like'.

There is a large body of work that shows us how to introduce **general conditional probabilities of the form $\text{pr}(A/B)$** where we have stabilized relative frequencies that support projection to new random sub-selections from the relevant class.^{xxii} These general conditional probabilities don't have the right form to play the role of guiding belief characterized in PP, precisely because they are *general* and *conditional*. PP requires something that is single-case and unconditional. For any given event – say the event that a particular coin toss comes up heads – there are indefinitely many general probabilities that apply to it. Choose any reference class to which the toss belongs and provided the class has the kind of microstructure that allows us to attach a probability, we will have the probability that coin tosses in that class come up heads, and they will not in general be the same. This is, of course, the old reference class problem, which PP avoids only because it is solved in the step from general probabilities to chances. First we find which of those general probabilities fixes the chance, and then we use the chances, via PP, to fix credence.

But once we have the general probabilities we can look to identify the **single-case, unconditional probability** of an event derived from the general conditional probability that can play that role, and so here we just look at what PP says. The PP says adopt *chances* as credences no matter what information you have from the past, provided you have no information from the future, so we look for probabilities that screen off all and only information from the past.^{xxiii}

There are two natural candidates:

- (i) The first goes naturally with a Lewisian framework in which theories of chance take the form of history-to-chance conditionals, the chance of e at t =^{xxiv} the general probability of an event like e following a pre- t history (where both e and pre- t history are characterized in intrinsic, or qualitative, terms). There are different ways that we might think about this quantity: e.g., as an expression of the information that history contains about e , or as a measure of the propensity of pre- t histories to produce e .
- (ii) The second fits more naturally with a physical setting in which we think of the intrinsic state of a system as determining the probabilities of events that fall in its future. In this case, we conditionalize not on all of history, but on the intrinsic state of the system in whose future e lies, so the chance of e for a system in a state S = the general probability of an e -type event for a system whose intrinsic state is S . As above, we can think of this quantity either as an expression of the information that S 's intrinsic state contains about whether e , or as a measure of the propensity of a system in S state to produce e .

If we assume a Markov condition, i.e., if intrinsic character of a system screens off any information from history, (i) and (ii) are quantitatively the same. Either of them will screen off historical information, but not information from the future (or, in a relativistic setting, information that is not drawn from the absolute past), and so either of them will be suited to playing the role of chance in guiding credence.

There are interesting connections between these. There's some very nice work in computational mechanics, for example, that shows that (under quite weak assumptions) we can start with a set of observable quantities, divide histories into equivalence classes that generate the same conditional probabilities for the observables. We can then use those to construct a Markov chain of what are called 'causal states' that screen off information from the past and generate the same conditional probabilities.^{xxv} What the causal states capture is all of the intrinsic structure in the system that is relevant to predicting the variables of interest. So there are lots of interesting things to say about the relationship between these two options, but in the special case in which states are causal states in the sense above, either of them will screen off all and only information from the past, and so either of them will be well-suited to play the role of chance in PP.

Finally, there are **credences**, which are subjective degrees of belief. These are descriptive of the epistemic states of believers.

What we have above are three logically distinct notions of probability: (i) the **general conditional probabilities**, which we can link to stabilized relative frequencies in reference classes, (ii) the **chances** (single-case unconditional probabilities that guide belief in accordance with PP), and (iii) the **credences**. We can say some things about relationships among them. The relationship between the general, conditional probabilities and the intrinsic state of the system to which they are assigned is metaphysically contingent, in the sense that it is mediated by laws that relate the intrinsic state of a system to its future behavior. We can think of the categorical properties as the bearers of chances, at least in a world in which the present chances pertaining to S supervene on the intrinsic state of S . The connection between general, conditional probabilities and credences is likewise contingent, mediated in this case by facts about our epistemic situation.

You shouldn't adopt chances as credences if you regularly get information from the future. They are good guides to belief only for creatures like us who don't have crystal balls, i.e., whose information about the future all comes by way of information about the present and past.^{xxvi}

In sum, we have a matrix of probabilistic notions, connected at one end to belief and at the other end to stabilized relative frequencies. Chance is linked to belief by PP, and general probabilities are linked to stabilized relative frequencies by the law of large numbers. ^{xliv} The internal links among elements in this matrix are uncontroversial except for the connection between chances and general probabilities^{xxvii}, which is constrained by the alignment of other elements.

VI The prospects for reduction

When chance is situated in this matrix, the closest connection that we get between chances and categorical facts comes from the link between general probabilities and stabilized relative frequencies. This secures an evidential relationship between frequencies and probabilities. But it also blocks reduction because it leaves open the *possibility* that the actual frequencies may diverge as far as you like from the probabilities.^{xlii} As I've said, I think that on this point, the Humean should concede that her account does not provide a reduction. The connection between probabilities and statistics *interpret* the probabilities without *reducing* them. Empirical content flows into the matrix through the connection between statistical probabilities and stabilized relative frequencies.

To acknowledge that the connection isn't tight enough to permit reduction, however, is not to commit one to a substantial anti-Humean ontology for chances. The Humean need only note that there is a difference between making statements about patterns in the mosaic, and making a guess, or venturing an inductive hypothesis based on those patterns. She can hold that the chances are best guesses based on patterns in the Humean mosaic, but deny that they are to be identified with those patterns. The difference matters because if we are trying to capture the *content of beliefs* about chance, then there are good reasons for denying that they are the same as beliefs about patterns in the Humean mosaic. They fail the modal test for sameness of content: we can hold the pattern of events fixed in our imagination and imagine that the chances vary. They are not intersubstitutable in inference: beliefs about chances license modal beliefs, which beliefs about patterns in the Humean mosaic do not. There is an ineliminable logical gap between beliefs about the chances and beliefs about the pattern of categorical fact that is codified in the axioms of probability and also essential to the role of beliefs about chance in guiding expectation. That looseness of fit makes inferences from patterns in the Humean mosaic to chances unavoidably *ampliative*.

The best systems analysis is best thought of not as an ontological reduction, but as a complicated inductive procedure that tries to extract patterns from what we know about the Humean mosaic and use them to guide credence about the future. This preserves the heart of the Humean idea that the modal structures that are best thought of as epistemic guides. The idea is that a physical theory is a kind of inductive machine that

guides prediction in the case of ignorance. One finds this idea in the writings of many Humeans. Carl Hoefer, for example, defending a Humean view with very strong affinities to the one here, echoes the idea that the chances are to be thought of as expressive of best guesses about unobserved cases based on stabilized relative frequencies.^{xxviii} He introduces as an example of a regularity, that

“A page of a high school yearbook containing row after row of photos of 18 year olds, in alphabetical order -- so that, in the large, there is a stable ratio of girl photos to boy photos on each page, say 25 girls to 23 boys). ... The regularity about boys and girls on pages is objectively there, and makes it reasonable to bet “girl” if offered a wager on the sex of a person whose photo will be chosen at random on a randomly selected page.” (Hoefer 2007, p. 18)

And of course he's right, the regularity is there in the world, as part of the mosaic of categorical fact, but the probability has inductive content that goes beyond the regularity. It bets on its persistence, commits itself to the expectation of its persistence, and has modal implications that outrun any mere belief in how things are. It is certainly no logical contradiction to notice the regularity and not associate it with a probability, i.e., not to take on the additional commitments that come with an assignment of probability. One can easily imagine situations in which that would be the rational thing to do (so, for example, if you are sitting at a roulette table and you notice that the apparently random behavior is actually carefully managed by a lever under the table). So what is 'really there' in the world is the regularity. The inductive content is a hedged prediction *based on* the regularity that carries epistemic and practical commitments that we make explicit by looking at its role guiding expectation and betting behavior.^{xxix}

If one goes back and looks at Lewis' papers, there are actually two separable threads in that paper: the one that treats chances as recommended credences and the one that identifies chances with patterns in the manifold of categorical fact.

My suggestion is that what the Humean view can be thought of as presenting the patterns in the Humean mosaic that provide the basis for inductive content built into the probabilistic belief. That inductive content doesn't make a full-on prediction about what will happen, but a hedged guess about how likely the alternatives are, given the system's intrinsic state and its history. The same goes for beliefs about laws, capacities, dispositions, and causes – i.e., all of the modal outputs of Best System style theorizing. These aren't first order beliefs about what is the case, but derivative quantities that encode inductive content based on patterns in the manifold of fact. Those quantities are *used* in epistemic and practical reasoning. Taking patterns in the mosaic and 'solving for the probabilities' is a way of preparing credences in advance for believers who find themselves with no direct source of information about the future but needing to make guesses. It is science doing what science does best: taking the everyday informal inductive practices and making an art out of them. Beliefs about chance are hedged predictions. They don't assert, “this will happen” or even “this will happen so and so many time, but a generalized form of prediction that I've called elsewhere a 'partially prepared solution to a frequently encountered problem'. They extract projectible regularities from

the pattern of fact and give us belief-forming and decision-making *policies* that have a general, pragmatic justification.

I propose that the right way to develop a Humean account is to hang onto the epistemic thread in Lewis's presentation as expressive of the content of statements about chance, and present the ontic part really as an account of the facts that ground the inductive guesses expressed by those statements. That accords quite well with the spirit of the Humean view.

VII The half-Humean view

This kind of account departs from the more familiar Humeanism in two respects: (i) it doesn't treat chance as the most basic form of objective probability, and (ii) it denies that beliefs about chances (and laws) are simply disguised beliefs about patterns in the Humean manifold. But it preserves the Humean ontology, remaining opposed to the reification of laws and chances as agents in the production of the phenomena. I would describe that view as giving a non-reductive analysis of modal *content* without inflating modal *ontology*. The half-Humean holds chances are inductions on stabilized relative frequencies that go beyond a mere *description* of actual frequencies in any form, and that we can tell the story about the emergence and function of probabilistic thinking without invoking anything but stabilized relative frequencies, and hence without invoking anti-Humean truth makers for probabilistic belief.

But the anti-Humean will say that the Humean misses the point. She will say that we don't just need an account of how we form those beliefs; we need an account of what *warrants* them. We need something in the world to ground the inductive inferences that the Humean says are expressed in beliefs about chances. She will say that our lives depend quite literally on the continuation of lawful regularities in their strict or probabilistic form, and hold that unless laws and chances are agents in the production of behavior, we have no reason to expect regularities to continue. This is really the crux of the dispute between the Humean and anti-Humean. Our physical theories provide us with models with a good deal of epistemic superstructure that guides limited creatures through a complex world. That is why we build theories and that is how we use them. Both sides agree on that. The debate between the Humean and anti-Humean is about whether that epistemic superstructure is *nothing more* than epistemic superstructure, or whether it represents modal structure that is part of the intrinsic fabric of the physical world.

IX Conclusion

The divide between the Humean and anti-Humean accounts is deeply entrenched, with many able defenders on both sides. While my own allegiance is to the Humean view, I think that what divides the Humean and anti-Humean are quite deep issues about where demands for explanation bottom out, and what counts as an explanation. The Humean says that we can explain the formation and function of beliefs about chance without supposing the existence of anything but stabilized relative frequencies in the pattern of actual fact. The anti-Humean asks 'what supports the stabilized relative frequencies?'. The Humean says 'all chains of

explanation end somewhere', or she says that if the anti-Humean thinks that simply postulating anti-Humean whatnots to keep the world running like clockwork really explains anything, she is wrong.... and (as Lewis might himself have said) so it goes.

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ⁱ The title of the article is a nod to Pearl 2001. An early version of this paper was presented at a workshop in London organized by Mauricio Suarez. Richard Pettigrew, Luke Glynn, Seamus Bradley, Roman Frigg, and Nina Emery were present and I benefitted greatly from the discussion. I would especially like to thank Guido Bacciagaluppi for years of discussion. I've learned an enormous amount from him, about chances and much else. Many thanks to Shamik Dasgupta for the invitation to contribute to this volume, and for the most gracious possible editorship.

ⁱⁱ There may be other reasons for challenging the Humean ontology (Maudlin 2007, pp. 78-103), but I put them aside here.

ⁱⁱⁱ It was Lewis who coined the term 'Humean' in this capacity after Hume, the great denier of necessary connections between distinct events. Whether the label is apt or not, it has stuck.

^{iv} Much of this literature has focused on formulations of the Principal Principle. The strategy has been to reformulate PP in a way that gets around the problem that Lewis put his finger on. A thriving industry, which now spans the literature in metaphysics and formal epistemology is devoted to these competing principles, which have acquired independence interest as chance-credence norms. See Hall 1994, Ismael 2008 and 2015b, Pettigrew 2015. The question of whether the Humean program could be reconciled with PP is widely regarded as having been settled in the affirmative, possibly with a little tweaking of the formal expression of PP.

^v See Callendar and Cohen 2009.

^{vi} Here's Lewis's own classic statement of the method, in its application to laws: "Take all deductive systems whose theorems are true. Some are simpler better systematized than others. Some are stronger, more informative than others. These virtues compete: An uninformative system can be very simple; an unsystematized compendium of miscellaneous information can be very informative. The best system is the one that strikes as good a balance as truth will allow between simplicity and strength. How good a balance that is will depend on how kind nature is. A regularity is a law IFF it is a (contingent) theorem of the best system." (1994, p.478)

The method was generalized by adding 'fit' to simplicity and strength to provide criteria for the choice between theories that include not only a set of laws but a theory of chance. So, says Lewis: "Consider deductive systems that pertain not only to what happens in history, but also to what the chances are of various outcomes in various situations - for instance the decay probabilities for atoms of various isotopes. Require these systems to be true in what they say about history.... Require also that these systems aren't in the business of guessing the outcomes of what, by their own lights, are chance events; they never say that A without also saying that A never had any chance of not coming about. (1995 p.480) The idea is that the higher the chance a system assigns to the true history (or to segments of it given part of the history) the better its fit.

^{vii} See Hall 2010 <http://plato.stanford.edu/entries/lewis-metaphysics/> for a nice discussion of this aspect of Lewis's account.

^{viii} See Loewer (2004), Callendar and Cohen (2009). The difficulty isn't merely to provide measures of simplicity, strength and fit, but to provide measures that *capture criteria operative in scientific theory choice*. Lewis was clear that he wanted the Best Systems Analysis to reproduce the epistemology of science. Scientists routinely use words like 'simplicity', 'strength', and 'fit' to describe extra-empirical criteria for theory choice, but resolving the vague and qualitative character of those criteria into something precise and quantitative has proven elusive. One option for the Humean is to reject the demand for general univocal notions of simplicity, strength and fit that can be formally defined and slotted in as criteria for theory choice. The informal notions of simplicity, strength and fit provide some guidance about the standards for choosing models, but they are loose enough to allow the details to get filled in by looking at scientific practice, and they allow for pragmatic trade-offs and some context- and problem-dependent choices of a kind one often finds in science. The pragmatic and metaphysically deflationary spirit of the Humean account allows for a little vagueness and variability in what the laws and chances are.

^{ix} Emery, Nina (2015) provides a recent defense of both of these objections.

^x Maudlin (2007) has pushed this objection with some force.

^{xi} Hajek (2012) contains a nice discussion of the difficulties of reduction.

^{xiii} Hall (this volume) mentions one. Maudlin (2007) presents one. Loewer (2004) responds that even though the simple world (S) is a model of our laws, our laws are not the laws of S. The laws of S are given by the best systematization of the facts at S. I don't think that the response is helpful. It strikes me that what these arguments are pointing to is that any (chance+law) package has a number of models and any Humean mosaic is a model of many different (chance+law) packages. This means that accepting a (chance+law) package comes with modal commitments that outrun any set of claims about the pattern of actual events. And that in its turn means that the content of a (chance+law) package is not exhausted by what it entails about the pattern of actual events. Whatever the nature of the inference from a Humean mosaic to a chance+law package is, I think that these arguments show that it falls short of reduction.

^{xiii} See Ismael (2015a).

^{xiv} There is another reason for wanting to do this; confusions generated by the way that the word 'chance' is used. In the tradition stemming from Lewis that conceives of a theory of chance as a collection of history-to-chance conditionals, and the chances as something derived from such a theory in a manner that is tailored to serve as a guide for belief. The philosophical tradition has mostly followed Lewis in using 'chance' to refer to the single-case probabilities. The foundational literature in physics uses 'chance' more loosely, sometimes in specific ways disambiguated by context, but often generically for any form of objective probability. And that usage has seeped into the philosophical literature in a way that produces confusion (e.g. Hoefer (1997, 2007) and Albert 2000).

^{xv} There is no compact simple rule or algorithm for statistical inference. In practice it is the canons of statistical inference and all of the formal and informal rules that are part of knowing how to apply them that relate probabilities to statistics.

^{xvi} This is an ill-defined, and somewhat open-ended defeasibility condition that is nevertheless workable in practice. We usually recognize selection procedures that defeat the expectation that the statistics in the sample will reflect those in the target, even if we can't provide an explicit criterion that covers all cases.

^{xvii} See Ismael (2008, 2015b), Hall (1994) Lewis (1994), Pettigrew (2015) for disputes about the proper formal expression of the principle. The discussion in this section draws on Ismael (2008 and 2011).

^{xviii} This substitutes Lewis' informal characterization of inadmissible information (as magical information from crystal balls and the like), with a physical characterization of inadmissible information as information about future events. In a relativistic setting, this is generalized so that inadmissible information is information not drawn from events in an agent's past light cone (see Ismael 2009 and Healey forthcoming a and b). Note that the generalization addresses one of Hall's objections to retaining the Lewisian view of chance as indexed to a time. Hall's framework for representing chances, which employs ur-functions, makes it easier to assimilate chance to familiar Bayesian reasoning, but it obscures the epistemic role of chance and makes it less easy to see how chances are distributed across space-time and related to the manifold of categorical fact. For that reason, I have retained the Lewisian framework.

^{xix} Hoefer (2007) offers a Humean view of general probabilities. He calls them 'chances', but he is using 'chance' to refer to any form of objective probability, whereas I reserve the term for the single-case probabilities that play the role of chance in PP. Since every particular event either happens or does not, it is at the level of general probability that we can match theoretical predictions to observed frequencies. The probabilistic predictions of the theory meet the observed frequencies in typical ensembles of the relevant reference class, but there is a loose fit between the general probabilities and observed frequencies for two reasons: (i) General probabilities bear a probabilistic relationship to frequencies in typical ensembles captured by the law of large numbers. A failure to match the observed frequencies can lower the likelihood of a theory, but never directly refute it, and (ii) we can have good systematic reasons for denying the assignment of a probability or assigning a probability different from an observed relative frequency.

^{xx} This is a reference to the complicated microstructure that a reference class has to have to support the attribution of a probability. Von Mises (1957) applies probability to what he calls *collectives* — hypothetical infinite sequences of attributes (possible outcomes) of specified experiments that meet the following requirements. A *place-selection* is an effectively specifiable method of selecting indices of members of the sequence, such that the selection or not of the index i depends at most on the first $i - 1$ attributes. There are two axioms:

Axiom of Convergence: the limiting relative frequency of any attribute exists.

Axiom of Randomness: the limiting relative frequency of each attribute in a collective ω is the same in any infinite subsequence of ω which is determined by a place selection.

The probability of an attribute A , relative to a collective ω , is then defined as the limiting relative frequency of A in ω . This kind of restriction, or one rather like it is needed to support the epistemic role of probability guiding belief about typical, not systematically chosen, subselections from the reference class. The von Mises criterion is quite strict. We can be less strict about the kind of stability that is required. The conditions we impose will tell us the conditions under which we can assign a probability and the inferences are licensed by that assignment. The stricter the conditions on application, the stronger the inferences that are licensed.

^{xxi}Ismael (2008) and Sober (2010).

^{xxii}Hoeyer (1997, 2007).

^{xxiii}Ismael (2011).

^{xxiv}The “=” here is an equality, rather than an identity.

^{xxv}Shalizi and Moore (2003). In that article, the authors are thinking primarily about coarse-grained states relevant to the prediction of a proscribed set of macrovariables, assuming an underlying microdynamics that fixes the evolution of the macrovariables. We extend it to fundamental theories by including all observables in the initial set of macrovariables.

^{xxvi}In a world in which we have information from the future, the chance-credence link is broken and so the chances provide no guidance.

^{xxvii}Proposed as a DEF in Ismael (2011).

^{xxviii}Here is his phrasing when he says what he shares with Lewis: “Objective chances are not primitive modal facts, propensities, or powers, but rather facts entailed by the overall pattern of events and processes in the actual world.” (Hoeyer, 2007, p.) On the strong reading ‘entailment’ means reduction: objective chances just are distributed patterns in the manifold of fact. On a weaker reading, they are recommended credences, based on pattern of fact.

^{xxix}And notice here that making the distinction between the general probability and the single case probability explicit is essential to getting the logic of the inference right. One assigns a general probability that makes explicit the expectation that the regularity persists, and there is an additional step that transfers that general probability to the single case and offers it as a recommended credence.